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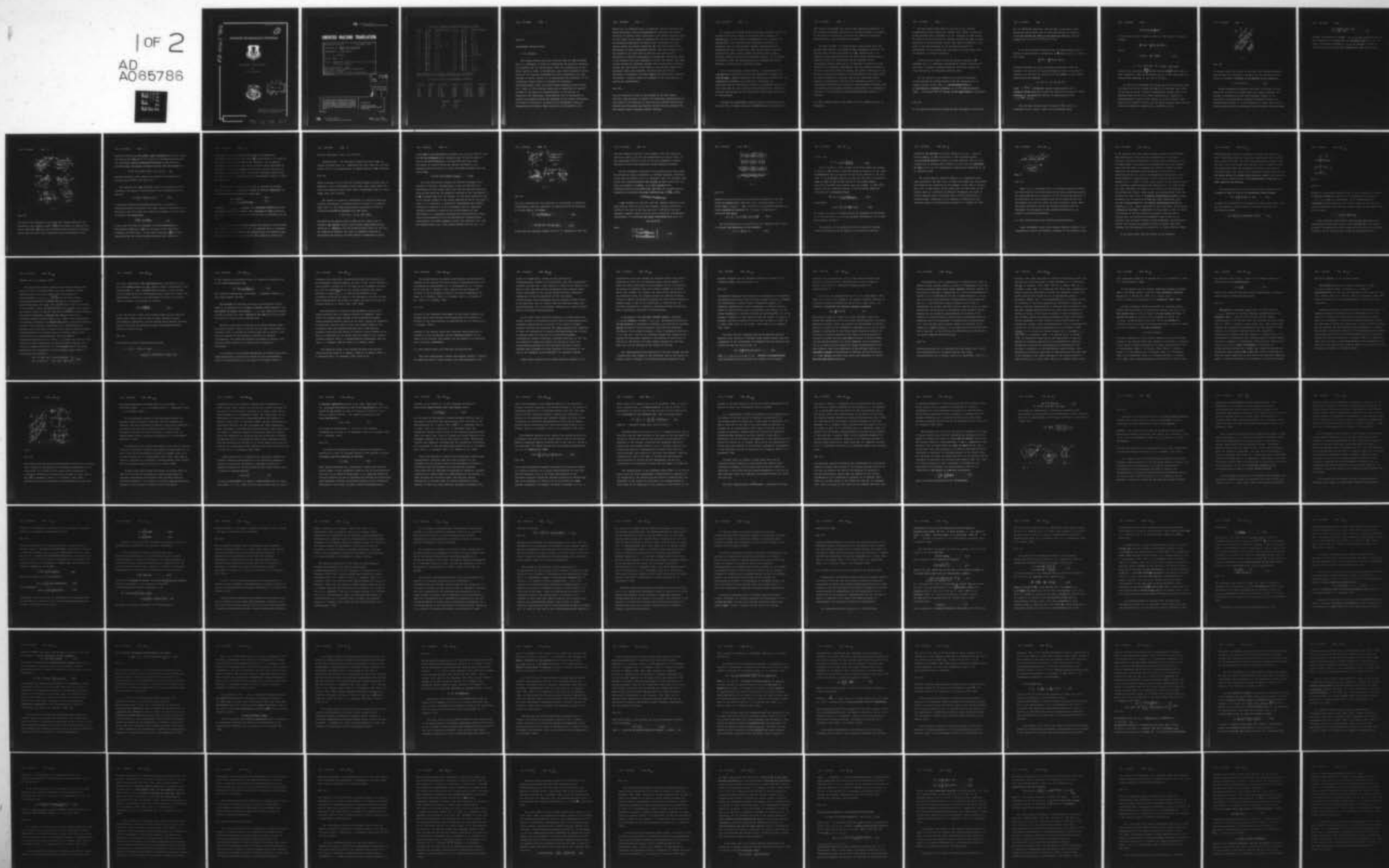

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HYDRODYNAMIC CASCADE THEORY

by

G. Yu. Stepanov

FOREIGN TECHNOLOGY DIVISION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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HYDRODYNAMIC CASCADE THEORY.

G. Yu. Stepanov.

The cascade theory will arise frcm the works of N. YE. Joukowski and S. A. Chaplygin, in which is investigated the action of turbines, cf propellers and the slotted wings. First are examined and are set forth, mainly in works in aerodynamics, some simple problems of plane motion of the inviscid incompressible fluid, generalizing the same problems of airfoil theory. It is simultaneous and independent of the aerodynamic cascade theory is developed the hydraulic (one-dimensional) theory of turbines whose beginning placed already by L. Euler in 1754, moreover appear and are permitted the separate problems of the theory of screw/propeller. In the forties in connection with appearance, investigations and the development of aircraft gas-turbine engines the beginning is the intense development the theory of lattices as of basis of the contemporary theory of compressors and turbines. Basic results will be obtained by the

school of N. E. Zhukovsy and S. A. Chaplygin and were connected with Moscow University, central aerohydrodynamic institute and central institute of aviation motor construction (here mention should be made of still works in the range of the hydraulic and steam turbines of the Leningrad polytechnic and Moscow power institutes, and also central boiler and turbine institute). In this basic stage of the development of theory hydrodynamic lattice they will become to call any located in the fluid flow or gas loop-type system of the motionless or rotating blade/vanes of turbomachine (hydraulic, steam or gas turbine, fan, vane compressor or pump). The defined thus space lattice includes as different special cases, single wing in infinite liquid, near the surface of the water or earth/ground; biplane and polyplane; paddle and propeller; the flat/plane and lattice; flat/plane, axisymmetric and three-dimensional/space ducts, channels and nozzles - actually almost all subjects of the investigation of applied gas hydrodynamics.

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From the theoretical point of the problem of the flow around lattices, they represent by itself the significant generalization of many tasks of the mechanics of fluid and gas, moreover during their solution were developed some specific methods and were obtained the new results, having sometimes overall interest.

At present the cascade theory will obtain universal acclaim and together with airfoil theory it will enter into textbooks on hydrodynamics, on the theory of turbomachines and on applied gas hydrodynamics. Its practical value continues to grow/rise in connection with the continuously expanded application/use of turbomachines and, in particular, with the creation of powerful hydroelectric power plants, thermal and atomic power plants, aircraft, rocket, stationary and transport engines - it is shorter, in connection with the application/use of turbomachines during obtaining of almost entire available energy.

For the contemporary cascade theory, is characteristic the complication of the models of flow and properties of liquid, use ETsVM [ЭЦВМ. - digital computer], the discussion of questions of the solvability of problems and stability of solutions, the application/use of contemporary methods of experimental studies. In this range work the many scientists and engineers from scientific research institutions and the educational institutions, which refer to turbomachines.

Although the hydrodynamic cascade theory is direct-connected with the flow of liquid and gas in turbomachines, representation of

this theory, its methods and results have application/appendices in all problems of periodic structures in continuous media, for example, in the theory of filtration, in acoustics, theory of elasticity, electrostatics, radio engineering and others.

The basic content of survey/coverage covers period from 1917 through 1967; however, in connection with fundamental value for the cascade theory of the early works of N. Ye. Joukowski and S. A. Chaplygin survey/coverage begins from these works, moreover here to introduce almost all designations and the concepts of the contemporary cascade theory and it is possible to outline the main trends of its subsequent development: from the simplest problems of the flow around the lattice of plates, the airfoil theory and of the cascade theories from fine/thin airfoil/profiles to the final theory of airfoil cascades of arbitrary form in the flat/plane steady potential incompressible flow with the subsequent account of the compressibility effects and viscosity. Survey/coverage concludes with two sections, which concern somewhat in more detail the contemporary problems of the unsteady and three-dimensional/space swelling of lattices.

§ 1. The cascade theory in the works of N. Ye. Joukowski and S. A. Chaplygin.

The most widely known model of the flow of the inviscid incompressible fluid through the infinite foil lattice of identical airfoil/profiles was introduced to N. Ye. Joukowski in 1890 during the study by it of the action of turbines and then was used in vortex conception of screw propeller (1912). This model is obtained in the plane of the scanning/sweep of the cylindrical section of turbomachine, if one assumes that the motion of liquid occurs over the surfaces of circular cylinders.

In works on the theory of wing and airfoil cascade, N. Ye. Joukowski and S. A. Chaplygin systematically utilize the vehicle of the theory of complex variable functions, giving the compact and ideal description of flat/plane potential flow.

Let the lattice of the identical and equally streamlined airfoil/profiles be arrange/located along the imaginary axis of complex variable z=x+iy (Fig. 1). **Incompressible flow is characterized by composite potential** $w = \varphi + i\psi$ **and by** composite **speed** $\bar{v} = v_x - iv_y = v \exp(-i\alpha)$ **=dw/dz, by the being analytic** functions z.

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In the depicted continuous circulation flow the domain of definition

w(z) is infinitely connected, function $\bar{v}(z)$ is periodical (with period it), and function w(z) is infinitely valued. At infinity before the lattice and after it the speed has different limiting values $\bar{v}(-\infty) = v_1 \exp(-i\alpha_1)$, $\bar{v}(+\infty) = v_2 \exp(-i\alpha_2)$.

If we use to analytic function $\bar{v}(z)$ the Cauchy theorem and to deform the duct/contour of integration L, as shown on Fig. 1, then we will obtain

$$\oint_L \bar{v}(z)\,dz = -\oint_C \bar{v}\,dz + \bar{v}_2 it - \bar{v}_1 it = 0$$

(contour integrals they pass in positive direction). After the separation of the real and apparent/imaginary parts of the written equality, are obtained the conditions of the absence in the flow of eddy/vortices and sources:

$$(v_{2y} - v_{1y})\, t = \Gamma, \quad v_{1x} t = v_{2x} t = Q, \quad (1.1)$$

where $\Gamma = \oint v_s ds$ - circulation around airfoil/profile and Q - volumetric fluid flow rate through the layer of the liquid of single thickness in one period of lattice. In noncirculatory flow $\Gamma = 0$ and $v_{2y} = v_{1y}$.

From the same theorem, used to function $\bar{v}^2(z)$, and S. A. Chaplygin's first formula (1910) for the composite force

$$\bar{R} = R_x - iR_y = \frac{1}{2} i\rho \oint_L \bar{v}^2\, dz,$$

of the acting per unit length of blade (ρ - the density of liquid), follows

$$\oint_L \bar{v}^2(z)\, dz = -\oint_C \bar{v}^2\, dz + \bar{v}_2^2 it - \bar{v}_1^2 it = 0,$$

whence

$$\bar{R} = i\rho\Gamma\bar{v}_{cp} \qquad \left(\bar{v}_{cp} = \frac{\bar{v}_1 + \bar{v}_2}{2}\right) \qquad (1.2)$$

or

$$R_x = \rho\Gamma v_{cp\,y} = \frac{1}{2}(v_2^2 - v_1^2)\rho t, \qquad R_y = -\rho\Gamma v_{cp\,x} = -(v_{2y} - v_{1y})\rho Q.$$

Formula (1.2) expresses the N. Ye. Joukowski theorum ~~against~~ about lift force of wing, obtained in 1906 and distributed by it in 1914 (by applying the theorem about momentum) in the case of lattice.

In 1890 N. Ye. Joukowski will supply and will solve historically the first task of the cascade theories of the jet-edge flow around the lattice of plates. Figures 2 schematically depicts flow in the physical plane z=x+iy and in those associating it. Lattice is arrange/located in plane z with period it. In the critical point S speed v=0. At points F_1 and F_2 (on the edges of plate) begin the free jets from constant on modulus with a velocity of $v=v_2$.

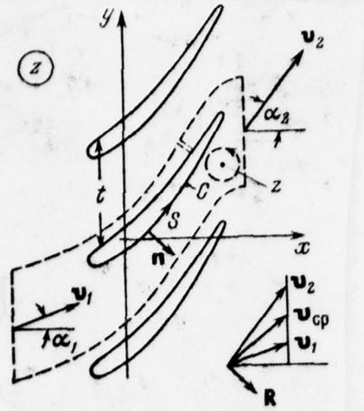

Fig. 1.

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At infinity after the lattice of jet, they reach maximum direction $\alpha=\alpha_2$ and pass at a distance σt (measured in the direction of the period of lattice). According to the equation of the continuity

$$v_{1x}t = (1-\sigma)\, v_{2x}t = Q. \qquad (1.3)$$

Unlike continuous circulation flow (Fig. 1) the zone of flow during the jet-edge flow around lattice is singly connected, the concept of circulation around airfoil/profile usually is not introduced, and respectively composite streaming potential w is single-valued function z. Of the projection of force, which acts on airfoil/profile, they are determined from the theorem about momentum:

$$\left.\begin{aligned}(v_{2x}-v_{1x})\,\rho Q &= (p_1-p_2)\,t-R_x,\\ (v_{2y}-v_{1y})\,\rho Q &= -R_y,\end{aligned}\right\} \tag{1.4}$$

mcreover on Bernoulli's integral $p_1-p_2={}^1/_2\,\rho\,(v_2^2-v_1^2)$, and pressure in the "stagnant" zone between jets is considered equal to p_2. In the plane of composite potential $w=\varphi+i\psi$ is obtained a series of semi-infinite cut/sections with carrying out (with period $it\bar{v}_1$).

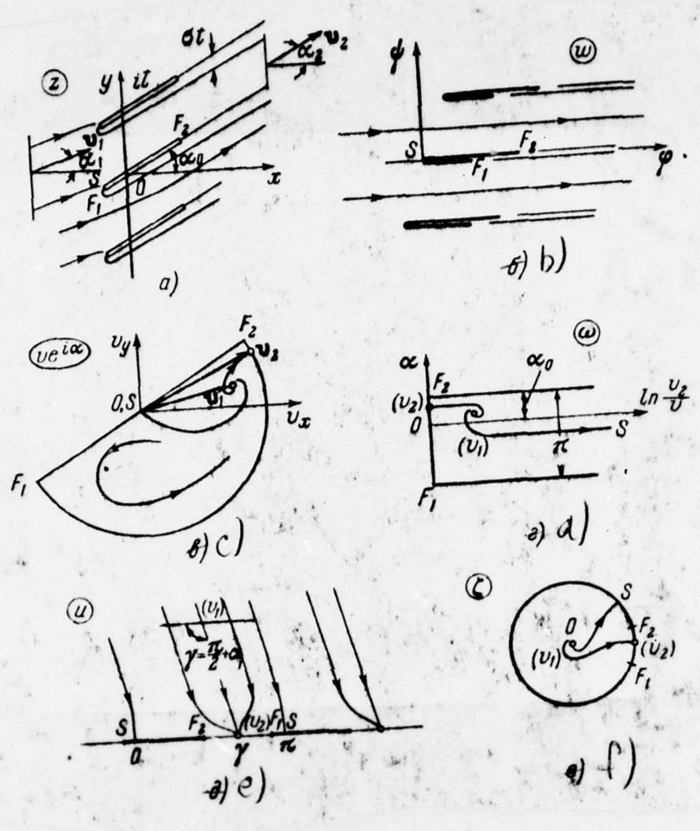

Fig. 2.

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The area of the hodograph of the speed (for clarity instead of the hodograph of the composite speed $\bar{v}=v\exp(-i\alpha)$ usually is depicted its mirror reflection $v\exp(i\alpha)$, that coincides with the speed in physical plane) takes the form of the semicircle, which corresponds to one

period of lattice. At the points, which correspond to infinity before the lattice and after it $(\bar{v}(-\infty) = \bar{v}_1$ and $\bar{v}(+\infty) = \bar{v}_2)$, are arrange/located the logarithmic special feature/peculiarities of the composite potential $w(\bar{v})$, the source of vortex and flow with the intensities

$$\Gamma_1 + iQ_1 = it\bar{v}_1 = tv_1 (\sin\alpha + i\cos\alpha), \quad Q_2 = -Q_1 = Q \qquad (1.5)$$

The flow parameters must satisfy the condition of coinciding the critical point $dw/d\bar{v}=0$ with point $\bar{v}=0$.

The developed by N. Ye. Joukowski method of the solution of the problems of the theory of jets consists in the construction of the function

$$\omega = \ln\frac{v_2}{\bar{v}} = -\ln\frac{v}{v_2} + i\alpha = \omega(u) \qquad (1.6)$$

in half-plane parametric alternating/variable u, to which is reflect/represented entire/all flow plane. After the construction of functions $\omega=\omega(u)$ and $w=w(u)$ the composite coordinate z of flow plane is found by the integration:

$$dz = \frac{dw}{\bar{v}} = \frac{1}{v_2} e^{\omega(u)} \frac{dw}{du} du. \qquad (1.7)$$

In this case the range of hodograph is reflect/represented to half-fringe $\alpha_0 > \mathrm{Im}\,\omega > \alpha_0 - \pi$, $\mathrm{Re}\,\omega > 0$ in the plane of the function of Joukowski, and flow plane - to the upper half-plane $\mathrm{Im}\,u > 0$ so that infinity before the lattice transfer/converts into infinity of

half-plane u, and all forms of plates and jet boundary are arrange/located along the real axis Im u=0 with period π. It should be noted that instead of parametric the variable u can be examined $\zeta=\exp 2i(u-\gamma)$, and then to one period of lattice will correspond the interior of single circle $|\zeta|<1$, but entire/all zone of flow will be reflected to infinitely leaved circle with logarithmic branch point in $\zeta=0$ ($z=-\infty$, $u=i\infty$). (Conformity of all characteristic points is shown on Fig. 2a-f).

Functions w(u) and ω(u) in the task in question are located through shvartsa - Christoffel's formula or they are constructed on special feature/peculiarities; in this case

$$\frac{dw}{du}=C_1\frac{\sin u}{\sin(u-\gamma)}, \qquad (1.8)$$

$$\frac{d\omega}{du}=\frac{C_2}{\sin u\sqrt{\sin(u-\beta_1)\sin(u-\beta_2)}}, \qquad (1.9)$$

Constants $C_1=-v_1t/\pi$, $C_2=-\sqrt{\sin\beta_1\sin\beta_2}$ are determined additionally from conformity conditions of planes. The assignment of three parameters β_1, β_2 and γ with an accuracy to scales (t and v_2) determines all the flow.

N. E. Joukowski in 1890 will obtain the solution of this problem for a special case $\gamma=0$ $\alpha_1=0$, $\Gamma_1=0$); it completed by S. A. Chaplygin and A. P. Minakov in 1930 with the republication of Joukowski's work. The solution of this problem obtained then again by A. Betz and E.

Peterson (Ingr-Arch., 1931, 2:2, 190-211).

Following task - the continuous circulation flow around the lattice of plates (Fig. 3) - completely they will solve for the first time in 1914 S. A. Chaplygin and, by another method, N. YE. Joukowski.

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Abroad a special case of this task (lattice without carrying out) is examined V. Kutta (Sitzungsber. Bayer. Akad. Wiss., Math.-phys. Kl., 1911), but complete solution will obtain considerably later E. Konig (ZAMM, 1922, 2:6, 422-429).

The lattice of plates is represented by a series of equal and parallel cut/sections, arrange/located with period it in physical plane (Fig. 3). The common/general/total properties of flow are establish/installed by formulas (1.1) and (1.2):

$$\Gamma = (v_{2y} - v_{1y})\,t, \quad v_{1x} = v_{2x}, \quad \bar{R} = -i\rho\Gamma\bar{v}_{cp}.$$

The speed in the leading edge of plate in the general case is infinite; the trailing edge of plate during adequate/approaching selection Γ coincides with the second critical point S_2, and then the speed in it is final. The plane of composite potential is one-sheeted and contains the semi-infinite cut/sections in which

coasts **when** $\varphi > \varphi_S$, **the velocity** potential has a gap to value Γ. The area **of the hodograph of** the composite speed $\bar{v}$ for one period of lattice occupies **half-plane**; at points $\bar{v}=\bar{v}_1$ and $\bar{v}=\bar{v}_2$, which correspond to infinity before the lattice and after it, are arrange/located the source of vortex and vortex discharge with the intensities

$$\Gamma_{1,2}+iQ_{1,2}=\pm\, itv_{1,2}(\sin\alpha_{1,2}+i\cos\alpha_{1,2}). \qquad (1.10)$$

For flow construction through the lattice of plates, it is possible to utilize a representation of zone of flow with the cut/sections on flow lines, which pass from the critical points S_2, onto the half-plane of the parametric variable um that introduced to N. Ye. Joukowski (see Fig. 2); however, in this case, unlike the case of jet stream, instead of the simple condition Re ω=0 on cuts F_1F_2 in plane u it is necessary to satisfy the considerably more complex condition of equality ω at the coinciding points of previously unknown cut/section in plane z. For the elimination of this difficulty, S. A. Chaplygin introduced this representation $u=u(z)$, that the duct/contours of all airfoil/profiles transfer/convert consecutively into equal segments $S_1S_2S_1'$ real axes $(S_1S_1'=2\pi)$, and the infinite point $z=\pm\infty$ - into points $u=ip+m\pi$ ($m=0, \pm1, \pm2, \ldots$).

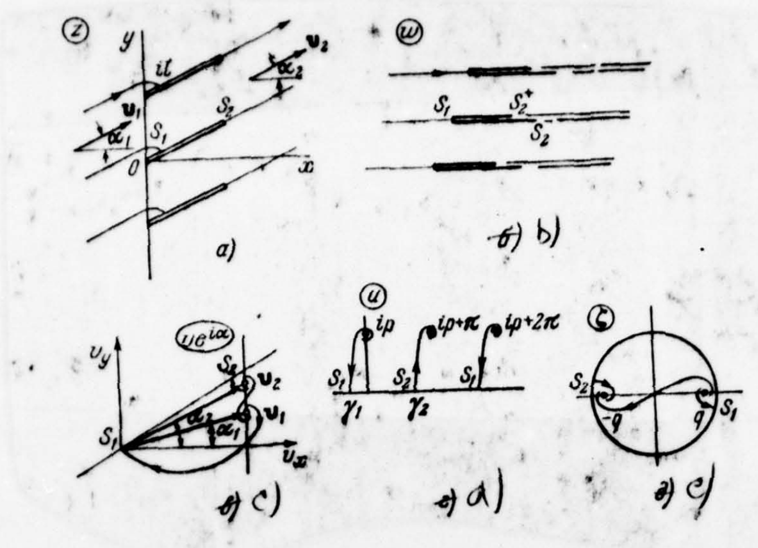

Fig. 3.

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With this representation the condition on cut/sections is fulfilled automatically. From the conformity of singular points and boundaries of planes u and z, it follows

$$\frac{dz}{du} = C\frac{\sin(u-\gamma_2)}{\sin(u-ip)\sin(u+ip)}. \qquad (1.11)$$

or

$$z = \frac{t}{\pi}\left(\ln\operatorname{ctg}\frac{u-ip}{2} + e^{2i\alpha}\ln\operatorname{ctg}\frac{u+ip}{2}\right). \qquad (1.12)$$

On the basis of conformal mapping (1.12) S. A. Chaplygin in 1914 will

give the complete solution of the problem of the flow around the lattice of plates, and with the republication of work in 1933, he will supplement function (1.12) by the free parameters, ensuring obtaining lattices from theoretical eight-parameter profiles.

All the subsequent solutions of the problem of the flow around the lattice of plates, instituted on conformal mappings, elementary follow from the solution of S. A. Chaplygin. Thus, for instance, most widely used representation of the interior of unit circle from the plane of parametric variable $\zeta = e^{iu}$ with symmetrically arranged/located branch points $\zeta=\pm q$ (see Fig. 3e) is obtained from Chaplygin's formulas by the simple substitution u=-i ln ζ, p=ln q:

$$z = \frac{2t}{\pi}\left(\operatorname{arth}\frac{q}{\zeta} + e^{2i\alpha}\operatorname{arth} q\zeta\right) + \text{const}.$$

N. Ye. Joukowski in 1914 will give the simplest solution of the same problem. first of all he will examine a special case of the lattice of the cuts of the real axis y=0, -1+mt<x<1+mt and will represent composite speed as sum of three velocities, corresponding longitudinal, to transverse and purely circulation flow (Fig. 4):

$$\bar{v}(z) = \bar{v}_{\parallel} + \bar{v}_{\perp} + \bar{v}_{o},$$

where

$$\left.\begin{aligned} \bar{v}_{\parallel} &= \bar{v}_{\parallel\infty} = \text{const}, \\ \bar{v}_{\perp} &= \bar{v}_{\perp\infty}\frac{\sin \varkappa z}{\sqrt{\sin \varkappa (z+1)\sin \varkappa (z-1)}}, \\ \bar{v}_{o} &= \frac{\Gamma\varkappa}{2\pi i}\frac{\cos \varkappa z}{\sqrt{\sin \varkappa (z+1)\sin \varkappa (z-1)}}. \end{aligned}\right\} \quad (1.13)$$

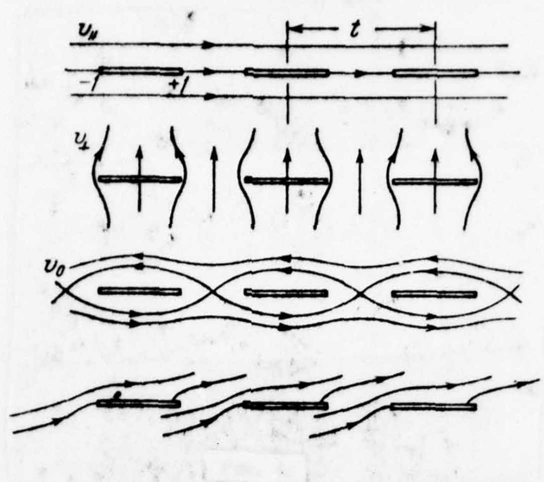

Fig. 4.

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Formulas (1.13) are written by analogy with formulas for the flow around the single plate, into which they transfer/convert in the extreme case $\varkappa = \pi/t \to 0$; their validity it is establish/installed by direct checking on boundaries of the region. Integration of expression $\bar{v}(z)$ gives

$$w(z) = \bar{v}_{\parallel\infty} z + \frac{\bar{v}_{\perp\infty}}{\varkappa} \arccos \frac{\cos \varkappa z}{\cos \varkappa} + \frac{\Gamma}{2\pi} \arcsin \frac{\sin \varkappa z}{\sin \varkappa}. \qquad (1.14)$$

For the finiteness of the velocity in the trailing edges z=1+mt, is necessary the observance of the condition

$$\bar{v}_{\perp\infty} \sin \varkappa + \frac{\Gamma\varkappa}{2\pi t} \cos \varkappa = 0; \qquad (1.15)$$

in this case

$$\bar{v}(z)=\bar{v}_{\|\infty}+\frac{\bar{v}_{\perp\infty}}{\cos\varkappa}\sqrt{\frac{\sin\varkappa(z-1)}{\sin\varkappa(z+1)}}. \qquad (1.16)$$

For the solution of the problem of the flow around any lattice of plates, N. E. Zhukovsiy utilizes composite potential of the simple noncirculatory flow through the lattice of the segments of one line $w(z)= w_{\perp}+w_{\|}$ as mapping function $z_1=w(z)$ and thus will be obtained in plane z_1 (Fig. 5) the oblique lattice of horizontal plates (establish/installed with carrying out) with period $T=t(\bar{v}_{\|\infty}+\bar{v}_{\perp\infty})$. Mapping function Joukowski assigns to the derivative (which is sufficient for calculations $\bar{v}(z_1)$)

$$\frac{dz_1}{dz}=\bar{v}_{\|\infty}+\bar{v}_{\perp\infty}\frac{\sin\varkappa z}{\sqrt{\sin\varkappa(z+1)\sin\varkappa(z-1)}}; \qquad (1.17)$$

respectively

$$z_1=\bar{v}_{\|\infty}z+\frac{\bar{v}_{\perp\infty}}{\varkappa}\arccos\frac{\cos\varkappa z}{\cos\varkappa}+\text{const}. \qquad (1.18)$$

The length of plates in oblique lattice is determined by difference z_1 at the critical points (S_1 and S_2) whose coordinates z are located from equation $dz_1/dz=0$.

The solution of the problem of the flow around the oblique lattice (arrange/located in plane z_1) is obtained in function

parametric the variable z; explicit solution is found only for special case $\bar{v}_{\|\infty}=0$, for the lattice of the horizontal plates, arrange/located without carrying out along imaginary axis y_1; this solution can be obtained from formulas (1.13)-(1.16) by replacement in them z by z_1, of all trigonometric functions by hyperbolic $\bar{v}_\perp$ on $\bar{v}_\|$ and vice versa.

The cascade theory, directly adjoins S. A. Chaplygin's work, which pertains to the year 1921, in which was studied the flow around the slotted wing, consisting of the segments of one line or circular arcs, and, in particular, it was shown, that for this wing, as for single, there is a parabola of stability (enveloping the lines of action resultant force of pressure flow) at final velocities on trailing edges. (Existence of the parabola of stability for the arbitrary system of airfoil/profiles proved M. V. Keldysh in 1936).

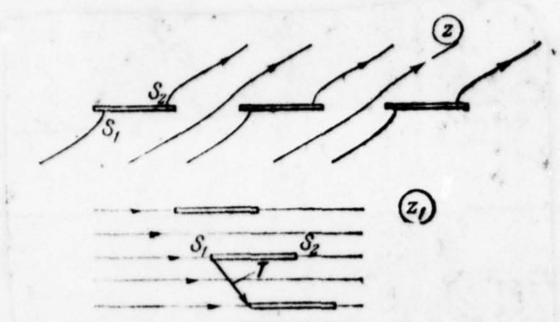

Fig. 5.

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Thus, to N. E. Joukowski and S. A. Chaplygin placed and solved the first tasks of the cascade theories and were establish/installed the common/general/total properties of the flow through lattices; in the mentioned, and also adjacent works on the theory of wing and hydrodynamics by them were shown the basic methods of study, which were being utilized by Soviet and foreign scientists with the subsequent the development of the cascade theories and in its technical application/appendices.

§ 2. The cascade theory from fine/thin airfoil/profiles.

Large development stage of the cascade theories consists in the propagation on them of the methods, developed in the twentieth years

in the theory of the flow around fine/thin Krylov airfoil/profiles. These methods will arise from the representation of the bound vortexes, distributed along airfoil chords, which, actually, will be the hydrodynamic interpretation of mathematical facts and formulas, known from the theory of potential and theory of complex variable functions. The systematic application/use of the latter is characteristic for the works of hydrodynamics specialists's Moscow school (author's survey/coverage of these works of distance M. V. Keldysh and L. I. Sedov in 1963). Basic results in the cascade theory from fine/thin airfoil/profiles were obtained by L. I. Sedov during the years 1935-1939. Its method of the solution of the problems of the slender-wing theory, *shock against* the incompressible fluid, the glidings and other tasks of flat/plane hydrodynamics consists in the systematic use of functions of form $g(z) = [\Pi (z - a_i)]^{\pm 1/2} \times [\Pi (z - b_j)]^{\pm 1/2}$ for the isolation/liberation of special feature/peculiarities in points $z = a_i$, $z = b_j$, which correspond to the wing edges, and obtaining solutions in the form of quadratures in flow plane. So will be solved the problems of the flow around thin wings in lattice without carrying out, the lattices of polyplanes, close to cuts of one straight line, the dual lattices of such polyplanes, arrange/located without the carrying out through the half of period, biperiodic lattices with the rectangle of periods (L. I. Sedov, 1939 and 1950).

In the basic first case the lattice of the fine/thin

airfoil/profiles, close to the lattice of plates with period πi, moves forward/progressively in plane z, moreover far before lattice ($z=-\infty$) liquid rests (Fig. 6). (This task unessentially differs from the task of the flow around rigid lattice, Fig. 1; however, it has some advantages during the propagation of method in the case of unsteady motion). The values of the composite velocity $\bar{v}(z)=dw/dz$ in the linear setting of slender-wing theory are carried to cut/sections $(-a<x<a,\ y=m\pi i)$: $\bar{v}^{+}=\bar{v}^{+}(x)$, $\bar{v}^{-}=\bar{v}^{-}(x)$, respectively in the upper and lower coasts of cut/sections.

Any disappearing with $z=+\infty$ periodic analytic function F(z) is expressed in zone of flow by the generalized Cauchy formula

$$F(z)=\frac{1}{2\pi i}\oint\limits_{L}F(\zeta)\,[\operatorname{cth}(\zeta-z)+1]\,d\zeta, \qquad (2.1)$$

in which the duct/contour of integration L, which covers point z, it can be expanded to infinity within the limits of period. For $F(z)=\bar{v}(z)$ this gives

$$\bar{v}(z)=\frac{1}{2\pi i}\int\limits_{-a}^{a}[\bar{v}^{+}(\xi)-\bar{v}^{-}(\xi)]\,[\operatorname{cth}(\xi-z)+1]\,d\xi. \qquad (2.2)$$

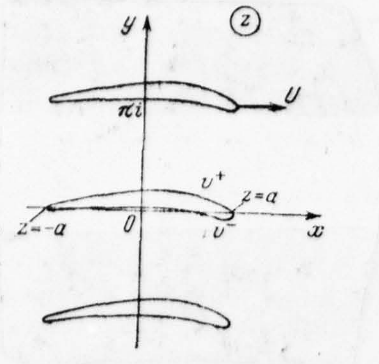

Fig. 6.

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Formula (2.2) expresses composite rate of flow, caused by the motion of lattice, through the value of the discontinuity of velocities $\bar{v}^{+}-\bar{v}^{-}=-(\gamma+iq)$ on cut/sections, and it hydrodynamically corresponds to the imposition of the flow, caused by the sources of vortex, distributed along all cuts $-a<x<a$, $y=m\pi i$, on the uniform flow with the speed

$$\bar{v}_{\infty}=-\frac{1}{2\pi i}\int_{-a}^{a}(\gamma+iq)\,d\xi=-\frac{\Gamma}{2\pi i}$$

During specified distributions $\gamma(x)$ and $q(x)$ (satisfying some supplementary conditions) formula (2.2) directly solves the inverse problem of determining the flow, caused by the lattice of theoretical profiles, and the construction of these airfoil/profiles (A. F.

Lesokhin and L. A. Simonov, 1939).

For more complex direct problem of the cascade theories with known the form of airfoil/profiles from the condition of their impenetrability, are known normal to the duct/contour of airfoil/profiles components of speed which in the first linear approach/approximation of slender-wing theory are assigned as apparent/imaginary parts $v_y^+(x)$ and $v_y^-(x)$ of function $\bar{v}(z)$ at the edges of cut/sections -a<x<a. In direct problem from formula (2.2), used to the boundary values of $\bar{v}(z)$ in the cut/section margin (or on duct/contour of airfoil/profile) ~~coasts~~ is obtained the integral equation of relatively unknown function $\gamma(x) = v_x^- - v_x^+$, equal to the discontinuity of tangential velocities. In slender-wing theory, known the numerical solution of this equation, instituted on the representation of function $\gamma(x)$ in the form of Fourier series with the isolated special feature/peculiarity on entering edge (G. Glauyert, Basis of the airfoil theory and screw/propeller, 1926; the Russian translation/conversion: M.-L., 1931). L. I. Sedov will give the final solution of direct problem in the form of definite integral. For this, the unknown function $\bar{v}(z)$ is divide/marked off on two parts, symmetrical and antisymmetric, $\bar{v}(z)=\bar{v}_1(z)+\bar{v}_2(z)$, that satisfy on cut/sections the conditions

$$\left.\begin{aligned} v_{1y}^+ &= -v_{1y}^- = \frac{1}{2}(v_y^+ - v_y^-), \qquad & v_{1x}^+ &= v_{1x}^- = \frac{1}{2}(v_x^+ + v_x^-); \\ v_{2y}^+ &= v_{2y}^- = \frac{1}{2}(v_y^+ + v_y^-), \qquad & v_{2x}^+ &= -v_{2x}^- = -\frac{1}{2}(v_x^- - v_x^+) \end{aligned}\right\} \quad (2.3)$$

The first, symmetrical, part hydrodynamically corresponds to flow from distributed sources $(q = 2v^+_{1y})$, and the second, antisymmetric, from eddy/vortices $(\gamma = -2v^+_{2x})$. Function $\overline{v}_1(z)$ directly is found through Cauchy formula (2.1). For determination $\overline{v}_2(z)$ taking into account the supplementary condition of the finiteness of velocity on trailing edges (z=-a+kπi) is introduced the function

$$g(z) = \sqrt{\frac{\operatorname{sh}(z-a)}{\operatorname{sh}(z+a)}}. \qquad (2.4)$$

In the cut/sections, coasts this function takes the pure imaginary values, which differ only in terms of sign; therefore product $F(z)=g(z)\overline{v}_2(z)$ immediately is found through Cauchy formula, moreover $\overline{v}_2(z)$ are obtained the necessary special feature/peculiarities, isolated by function g(z).

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The solution of direct problem takes the form

$$\overline{v}(z) = \frac{1}{2\pi}\int_{-a}^{a}(v_y^- - v_y^+)\left[\operatorname{cth}(\xi - z) + 1\right]d\xi + \\ + \frac{1}{2\pi i g(z)}\int_{-a}^{a}(v_y^- + v_y^+)\, g^-(\xi)\left[\operatorname{cth}(\xi - z) + 1\right]d\xi. \qquad (2.5)$$

To this solution in the general case, is reached the composite rate of the purely circulation flow

$$\bar{v}_{\circ}(z)=\frac{\Gamma}{2\pi i}\frac{\operatorname{ch} z}{\sqrt{\operatorname{sh}(z+a)\operatorname{sh}(z-a)}} \tag{2.6}$$

with arbitrary supplementary circulation Γ (moreover velocity on rear edges becomes infinite).

For lattices of infinitely fine/thin airfoil/profiles (small arcs) $v_\psi^- = v_\psi^+$ and the first integral in formula (2.5) disappears; for the lattice of plates, furthermore, $v_\psi^- = v_\psi^+ = \text{const}$, the second integral is taken and gives the exact solution of the task of the flow around the lattice of plates without carrying out.

Analogous solutions are obtained in all other examined tasks. For airfoil cascade, close to the cuts of real axis, the nucleus of integrals (2.5) is replaced on [ctg(ζ-z)-tga] and in formulas (2.4) and (2.6) instead of the hyperbolic functions are obtained trigonometric; for biperiodic lattices in nucleus and function g(z) they enter with respect to ζ- function and σ- function of Weierstrass.

On the basis of the obtained expressions of velocity, were given common/general/total formulas for forces and the torque/moments, acting on airfoil/profile. In the case of the oblique lattice of

fine/thin airfoil/profiles (establish/installed with carrying out) are recommended conformal mapping (of type (1.18)) for the lattice of plates without carrying out in the plane of the parametric variable $z_1=\zeta$ and the representation of the unknown functions with series according to degrees exp ζ. Is shown, the possibility of the propagation of method the cases of the presence in the flow of point dipole singularities and sources of vortex, and also of jet-edge and unsteady flows (L. I. Sedov, 1938, 1939, 1950).

The used method of study has high fundamental value, being direct-connected with the complete solution of Keldysyu - Sedov (1937) the boundary-value problem of the construction analytic complex variable function, taking on the cuts, filling the boundaries of half-plane (circle, band or ring), the assigned values of the alternately real and apparent/imaginary part of the function. Analogous method was used during the solution of the problems of the theory of elasticity and it entered into the theory of the singular integral equations (see N. I. Muskhelishvili's monographs, 1946 and 1962; S. G. Mikhlin, 1949 and 1963; F. D. Gakhova, 1963).

The particular tasks of the cascade theories from fine/thin airfoil/profiles solve B. A. Mayzel' (1936), P. A. Walter (1938), I. M. Belen'kiy and I. E. Zelenskiy (1938, 1939) et al.

The cascade theory in question from fine/thin airfoil/profiles obtained in the thirties the first practical application/uses in the calculations of hydroturbines (I. N. Voznesensky and V. F. Peking) and of fans (K. A. Ushakov, 1936); then it was used during the creation of the contemporary theory of axial-flow compressors and pumps (G. F. Profkur, 1934; L. A. Simonov, 1957; K. A. Ushakov et al., 1960; A. A. Lomakin, 1950).

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In spite of the subsequent development of more general methods, the cascade theory from fine/thin airfoil/profiles until recently is utilized in the investigations of hydroturbines (A. Yu. Kolton and I. E. Etinberg, 1958) [1].

FOOTNOTE [1]. The cascade theory from fine/thin airfoil/profiles is utilized in some contemporary foreign management/manuals on the theory of the lattices (for example, see the works of G. Schlichting and P. Shol'ts). ENDFOOTNOTE.

§ 3. The cascade theory from arbitrary airfoil/profiles.

The first investigations concern the simplest objects - lattices of plates and close to them. However, the airfoil/profiles of the

blades of turbomachines, unlike the wing profiles and screw/propellers, usually are strongly bent, they have considerable curvature and work over a wide range of a change in the angles of the entry of flow. Therefore, large effort/forces were directed toward further development of the cascade theories without substantial limitations of form and conditions of the flow around airfoil/profile. This development rests on results, reached earlier in the particular case of lattices from the fine/thin weakly bent airfoil/profiles, and on the methods, developed in a precise airfoil theory of arbitrary airfoil/profile.

To the works Soviet scholarly according to airfoil theory and airfoil cascades in flat/plane potential flow the characteristically systematic application/use of methods of the theory of complex variable functions for explaining the common/general/total properties of flow, its construction on special feature/peculiarities is direct in physical plane and with the use of conformal mappings, representation analytic functions, connected with flow, in the form of integrals or series and finally the solution of direct and reverse/inverse problems of the flow around lattices as basic boundary-value problems for these functions in flow plane, in the area of the hodograph of the velocity or in canonical regions.

Basic direct problem of the cascade theories consists in the

construction of the flow through the assigned lattice under given conditions of flow; all the remaining tasks, usually simpler, in which the lattice previously is not assigned, but it is obtained in the process of solution, they call inverse problems in the broad sense. They include, for example, the tasks of the construction of the continuous either jet streams through the lattices of plates, circles or special theoretical profiles, and also inverse problems in narrower sense - task of the construction of lattices with the specified distribution of velocity, satisfying specific conditions, and, in particular, assigned on airfoil/profile.

As analytical functions were utilized composite potential $w = \varphi + i\psi$, composite velocity $\bar{v} = v_x - iv_y$, its logarithm $\ln\bar{v} = \ln v = i\alpha$ - (or the function of Zhukovskiy $\omega = \ln(\bar{v}/v_2)$), the function of conformal mapping of zone of flow from physical plane with the composite coordinate z onto canonical ranges in plane of parametric variable u, ζ or Z (see Fig. 3 and 7). As canonical regions most frequently are applied the half-plane, exterior or the interior of circle and the exterior of the lattice of plates without carrying out, and also band, ring and the exterior of the lattice of circles.

The common/general/total properties of the flow through lattices coincide with those studied in the particular case of the lattice of plates, which is obvious, if we examine conformal mappings of the

assigned lattices onto the equivalent lattices of plates or the canonical ranges, depicted in Fig. 3-5.

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Is especially important the property of the linearity of the tasks of determining the functions w(z) and $\bar{v}(z)$, because of which is possible the construction of complex flows by imposition simpler, the elementary recalculation of the distribution of velocity during a change in flow conditions and, in particular, the existence of the linear dependences of relative velocity on airfoil/profile and of the tangent of angle α_2 of flow discharge from the tangent of angle α_1 of entrance, and also circulation from the sine of angle of attack (L. I. Sedov, 1939, 1950; N. Ye. Kochin, 1944, 1949; G. Yu. Stepanov, 1953, 1962).

The basic integral representation of the periodic analytic functions F(z) (limited at infinity) gives Cauchy integral with the propagation of the duct/contour of integration for the period of the lattice (as this was done in Fig. 1):

$$F(z) = \frac{1}{2\iota i} \oint_{C} F(\zeta) \operatorname{cth} \varkappa (\zeta - z) \, d\zeta + F_\infty. \qquad (3.1)$$

Here $F_\infty = {}^1/_2 [F(-\infty) + F(+\infty)]$, $\varkappa = \pi/t$, lattice is arrange/located along imaginary axis with period T=it. Formula (3.1) actually

coincides with representation (2.1) of the cascade theories from fine/thin airfoil/profiles, differing from it only in terms of overall duct/contour of integration during transition to airfoil/profile C.

On the basis of representation (3.1) N. Ye. Kochin (1941, 1947, 1949) will solve direct problem of the cascade theories, applying the reception/procedure, developed by him in 1937 in wave theory. For solution is introduced the function (integral transform)

$$H(\mu)=\oint_C \bar{v}(z)\,e^{-i\mu z}\,dz, \tag{3.2}$$

relatively by which of formula (3.1) with $F(z)=\bar{v}(z)$ taking into account the conditions of the flow around airfoil/profile is obtained functional equation. After the solution of this equation of function $\bar{v}(z)$ it is located by the inversion of conversion (3.2). As the first example of application of his method Kochin obtained a precise (in the form of a series) solution of the task of the flow around the lattice of circles and then approximate solution of direct problem for the arbitrary lattice of a small denseness (on the assumption that it is known the flow around the single airfoil/profile of the same lattice). Upon this formulation of the problem, directly is explained a change of distributing of velocity and its circulation $(\Gamma=H(0))$ during transition from single airfoil/profile to lattice from the same airfoil/profiles.

Representation (3.1) in application to functions $w(z)$, $\bar{v}(z)$ or $\ln\bar{v}(z)$ on basic cascade profile L_0 after the compartment of real and apparent/imaginary parts gives linear integral equations relative to the velocity potential φ, projections v_x, v_y or the module/modulus of velocity v as of functions of the arc of airfoil/profile s. In the case of lattices from fine/thin airfoil/profiles, these equations have in § 2 the effective solution indicated in the form of quadratures; for profile arbitrary form of equation, they are solved numerically, by the method of information to the system of linear equations or by successive approximations. This method of the solution of direct problem is called usually vortex/eddy method in connection with the hydrodynamic interpretation of representation (3.1) with $F(z)=\bar{v}(z)$ and in another manner the obtaining of the equations of task as a result of the imposition of the uniform flow with a speed of $\bar{v}_\infty$ on flow from the eddy/vortices, distributed on the duct/contours of airfoil/profiles.

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Vortex/eddy method, as in principle idle time itself, will receive wide acceptance and it is applied both for the single airfoil/profiles (P. A. Walter, 1922; M. A. Lavrentyev, 1932; A. P.

Melnikov, 1936, 1949, etc.) and for lattices from fine/thin small arcs and from the airfoil/profiles of final thickness (P. V. Melent'yev, 1939; A. F. Lesokhin, 1940, 1946, 1953; V. V. Uvarov, 1946; M. I. Joukowski, 1949, 1960). In connection with large labor consumption of method, obtaining sufficiently accurate and systematic results will become possible only during the use of contemporary computer technology (ETsVM). By S. M. Belotserkovskiy, A. S. Ginevskiy and Ya. Ye. Polonskiy in 1962 were published the calculations (using the method of discrete eddy/vortices) of lattices from biparametric small arcs with maximum sagging/deflection to 30o/o and his position to 30-50o/o of chord. On the basis of results of these calculations, were obtained useful interpolation formulas for basic hydrodynamic lattice parameters of those utilized in axial-flow blowers and compressors. Somewhat later vortex/eddy method will be programmed and was used in the practical calculations of the lattices of steam turbines and stationary gas turbine engines (M. I. Joukowski, N. I. Durakov and G. I. Novikova, 1963; V. M. Zelenin and V. A. Shilov, 1963). In theoretical sense and for the application of numerical methods are important questions of the solvability of equations, convergence of successive approximations and evaluation of the accuracy of solutions. In the hydrodynamic cascade theory, these questions are theory of elasticity in connection with the close tasks of the voltages in the plane, weakened by the infinite series of equal grooves (G. N. Savin, 1939, 1951; S. G. Mikhlin, 1949) and by

their biperiodic system (L. M. Kurshin and L. A. Fil'shtinskiy, 1961; L. A. Fil'shtinskiy, 1964).

In the special case of circular, generally speaking, revolving gates, vortex/eddy method is applied after preliminary conformal mapping (P. V. Melent'yev, 1939; V. V. Uvarov, 1946) and it is direct in flow plane (V. P. Lukashevich, 1964, 1965).

Another widespread reception/procedure of conformal mapping reduces direct problem of the flow around lattice (Fig. 7a) to the more studied task of flow in simply connected region. For this, is applied the periodic function, most frequently $\zeta = \exp 2\varkappa z$ or $\zeta = \mathrm{cth}\, \varkappa z$. In the obtained range (Fig. 7b) it is necessary to construct flows from special feature/peculiarities - source of vortex and vortex discharge - with the intensities

$$\Gamma_{1,2} + iQ_{1,2} = \pm t v_{1,2} e^{i\alpha_{1,2}},$$

for which necessary to solve integral equation or to find conformal mapping of the obtained range into canonical. The latter is conducted by the special methods, developed for a single airfoil/profile (Ya. M. Serebriyskiy, 1944; L. A. Simonov, 1945, 1947; S. G. Nuzhin, 1947), or it is numerical (G. M. Goluzin, 1947; P. V. Melent'yev, 1937; M. A. Lavrentyev, 1946; L. V. Kantorovich and V. I. Krylov, 1941, 1949). When conformal mapping of the exterior of lattice onto

the canonical range Z=Z(z) is found or it is known, velocity is calculated by the differentiation:

$$\bar{v}(z) = \frac{dw}{dZ} \frac{dZ}{dz}, \tag{3.3}$$

moreover the composite potential w=w(Z) easily stands in canonical range on special feature/peculiarities.

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The method of conformal mapping with the aid of function $\zeta = \mathrm{cth}\,\varkappa z$ and after the blowing numerical representation on the **average in plane** Z (Fig. 7c) will be basic for the foreign investigations of the fortieth years (A. Khauyell, ARS Techn. Rept, 1941, No. 5095; V. Traupel', Sulzer Techn. Rev., 1945, one-to-one). However, this sequence of representation for the lattices of large denseness is virtually unsuitable due to very nonuniform representation (|dZ/dz|) and with respect to a larger error in the determination of velocities. This deficiency/lack is removed during the use of a sequence of conversions ζ_1=a+bζ with the transition of points ζ_1=±1 to the **centers of curvature** of the edges of airfoil/profile and then $\zeta_2 = \varkappa^{-1}\,\mathrm{arcth}\,\zeta_1$; as a result is obtained the simple oval range (dotted **line in Fig. 7a)**, close to vane channel. Then this range numerically or with the aid of electrical simulation (EGDA) is reflect/represented to circumference or band with smaller

oscillation |dZ/dz| (G. Yu. Stepancv, 1962).

For circular gratings are applied conversicns $z_1 = \ln z$ or $z_1 = z^{1/N}$ ($z = 0$ in the center cf lattice, N - a number of airfoil/prcfiles) and then the same procedure as for usual lattices (P. A. Walter, 1925; V. V. Uvarov, 1946; G. I. Maykapar, 1949, 1952); dcuble-row lattices are reflect/represented to ring or band with cut/section.

Method EGDA widely is utilized in Soviet and foreign works. It replaces laborious calculations by the measurements of electric potential on models from electrolyte or the carrying out paper. Eefore the development cf sufficiently effective numerical methods and appearance of ETsVM, this method will be cnly, virtually suitable for any lattices. The most advisable applicaticn/uses of a method EGDA ccnsist in the determination of the ncncirculatory flow through the assigned lattice for its conformal mapping onto the equivalent lattice of plates or circles (L. A. Simcnov, 1940; S. F. Abramovich, 1946) and it is later in the determination cf the representation of interior of the type of those given in planes ζ or ζ_2 (Fig. 7) or the hodcgraph cf velocity in canonical region.

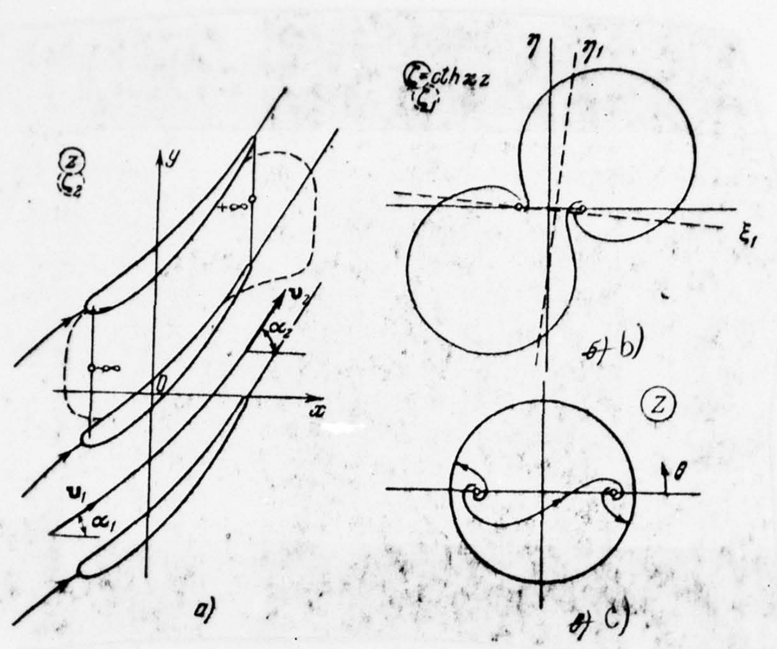

Fig. 7.

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The simplest and most precise render/showed the method of "division", which consists in determination by elementary measurements on the duct/contour of the model of the points, which correspond to consecutive indexings of circular arcs (in plane ζ in Fig. 2f) in half (A. M. Luxemburg, 1949; G. Yu. Stepanov, 1950, 1957). Presentation of those mentioned and survey/coverage of some other

application/appendices of method EGDA are in the works I. M. Tetel'baum (1952), P. F. Fil'chakova and V. I. Panchishin (1961), G. F. Stepanov (1962).

All the numerical methods of the flow-field analysis the assigned airfoil cascades of arbitrary form are laborious and, actually, give only approximate solutions of particular character. In connection with this were developed and will receive known propagation theoretical airfoil cascades of special form, the allow/assuming precise analytical expressions of the hydrodynamic functions of flow.

By generalizing the Schwarz-Christoffel in the case of latticed range were constructed lattices from symmetrical quadrangles (E. L. Bloch, 1947) and from arbitrary triangles in straight line and in circular gratings (D. A. Voytashevskiy, 1953, 1956). This same by method in principle it is possible to obtain lattices from polygons with the arbitrary number of sides (L. I. Sedov, 1950).

In many theoretical studies the lattice of circles plays the same role as circle in the theory of single airfoil/profile; therefore considerable effort/forces were directed toward the flow-field analysis of the lattice of circles. Common/general/total approach to the solution of problem for the arbitrarily

arranged/located final system of circles will be indicated N. V. Lambin (1934, 1939). Precise (in the form of a series) solution for the lattice of the circles of distance N. E. Kochin (1941) and, in another manner, G. S. Samoylovich (1950). The calculations of the distribution of velocity by vortex/eddy method will fulfill B. L. Ginsburg (1950) and A. I. Borisenko (1955). For the construction of theoretical lattices, are more convenient the final expressions, which correspond to the flow around the lattices of some ovals, close to circles. A classical example of such ovals, which are obtained during the imposition of the steady flow on the lattice of dipoles, was improved via the replacement of point dipole distributed along some axis intercepts of lattice (E. L. Bloch, 1947), also, with the addition of one additional point dipole in the center of this cut (E. L. Bloch and A. S. Ginevskiy, 1949, 1953).

Great possibilities of obtaining the theoretical lattices of distance different generalizations of the theoretical Zhukovskiy profiles and Chaplygin. According to Joukowski's method, the composite potential of the noncirculating flow around single circle is applied as mapping function

$$z = w(\zeta) = \zeta + \frac{1}{\zeta} \tag{3.4}$$

to certain duct/contour C in plane ζ, which covers point $\zeta=0$ whose flow around $w_C = w_C(\zeta)$ known or can be easily found. Then in plane z

is obtained theoretical profile $C_1(z_{C_1}=z(\zeta_{C_1}))$ with known flow $\bar{v}(z)=(dw_C/d\zeta)(d\zeta/dz)$. For obtaining one of the generalizations of method instead of the circle, is taken the lattice of circles or close ovals, as mapping function - the composite potential of its noncirculating flow

$$z=w(\zeta)=\zeta+f(\zeta), \tag{3.5}$$

and instead of duct/contour C - lattice of the displaced circumferences or ovals (S. F. Abromvoch, 1950; L. A. Dorfman, 1952; M. I. Joukowski, 1960).

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According to Chaplygin's method, theoretical profiles are obtained as a result of conformal mapping of the exterior of circle from plane ζ by the conversion of the form

$$z=\zeta+F(\zeta), \tag{3.6}$$

where analytic function F(ζ) disappears at infinity and contains certain number of free parameters, which lay out of airfoil/profile; with F(ζ)=1/ζ is obtained single plate. S. A. Chaplygin himself in 1933 will indicate the way of obtaining the lattices from theoretical eight-parameter profiles, generalizing function (1.12), reflecting half-plane to the lattice of plates. Another generalization is

obtained, if we instead of $1/\zeta$ take analogous according to hydrodynamic sense function $f(\zeta)$ from formula (3.5):

$$z=\zeta+F\left(\frac{1}{f(\zeta)}\right). \quad (3.7)$$

As F is taken the biparametric linear-fractional function, used by Chaplygin for obtaining the theoretical profiles, and some of its generalizations (E. L. Bloch, 1947, 1951; A. S. Ginevskiy, 1951; E. I. Umnov, 1952; E. L. Bloch and A. S. Ginevskiy, 1961), the calculations showing that the form of the obtained airfoil/profiles (with the center line, close to circular arc) weakly depends on the geometric parameters of lattice. The results of these calculations together with numerical investigations are utilized for the shaping of the lattices of fans and axial-flow compressors (A. S. Ginevskiy, 1953, 1961; S. A. Dovzhik, 1958; K. A. Ushakov et al., 1960).

Many other examples of lattices from theoretical profiles they supply/deliver, beginning with the first solution of N. Ye. Joukowski, the jet streams which can be considered as lattices of half-bodies with the hardened jets, and also the lattices, constructed with the specified distribution of velocities. Actually construction is always known the representation of the exterior of these lattices onto canonical range, and they have certain distribution of velocity under the design conditions of flow, changing it under any other conditions according to formula (3.3).

after the development of the effective methods of the solution of straight line and, especially, the inverse problems of the lattice of theoretical profiles to a considerable degree, they will lose their practical value, remaining, however, standard for estimating the accuracy of the approximate and numerical methods, and also for the construction of good first approximation or basic part of mapping function during calculations the flows around close lattices (L. A. Dorfman, 1962, N. N. Polyakhov, 1952; V. P. Cheprasov, 1958, etc.).

The effective solution of all tasks of the cascade theories can be obtained with the aid of the second general idea of the periodic analytic limited at infinity function F(z) in the form of a series on the derivative $\operatorname{cth} \varkappa z$, which follows from its integral representation (3.1) (G. S. Samoylovich, 1950):

$$F(z)=\varkappa\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}c_{-(n+1)}\frac{d^n}{dz^n}\operatorname{cth}\varkappa z+F_{\infty}. \qquad (3.8)$$

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This series generalizes expansion according to the negative degrees of z analytic function out of single airfoil/profile and coincides with the degenerate case of the known representation of the biperiodic functions through the derived ζ-functions of Weierstrass, what is the advantage of series (3.8) in the relation to other possible expansions, for example, according to degrees $\exp \varkappa z$ (L. I.

Sedov, 1950) or to degrees $\operatorname{cth} \varkappa z$ (N. N. Polyakhov, 1959). In circle $|z|<t$, function F(z) is decompose/expanded in Laurent series, the coefficients of correct part of whom by known form are expressed as the coefficients of its principal part (and of a series (3.8)):

$$F(z) = \sum_{n=1}^{\infty} \frac{c_{-n}}{z^n} + F_{\infty} + \sum_{n=0}^{\infty} z^n \sum_{k=n+1}^{\infty} \frac{(-1)^{n+1} 2^k B_k c_{-(k+n)}}{k(k-n-1)!\,n!} \varkappa^k, \qquad (3.9)$$

where B_k - Bernoulli number $B_0=1$, $B_1=0$, $B_2=1/6$...).

Utilizing series (3.8) and (3.9), G. S. Samoylovich will give in 1949 idle time the exact solution of the tasks of the flow around the lattice of circles and lattice from arbitrary airfoil/profiles as expansion of the function, reflecting the equivalent lattice of circles into the assigned lattice, in N. E. Kochin's setting, i.e., considering known the representation of single circle onto its airfoil/profile. This solution in the ideal form answers a question concerning the role of the angle of setting the fixed/recorded airfoil/profile in lattice and its densenesses, and also the constructions of theoretical lattices with any number of parameters.

The representation of the composite velocity $\bar{v}(z)$ in the form of series (3.8) has clear hydrodynamic interpretation as imposition of the velocities of the uniform flow and lattice of eddy/vortices and multipoles of all orders. In particular, the necessary number of first terms of the expansion of this velocity in the solution of the

problem of the flow around the lattice of circles corresponds to the lattice of ovals, how conveniently close to circles.

G. S. Samoylovich's results are utilized for the construction of the tables of the flow around the lattice of circles with the controlled/inspected accuracy over a wide range of densenesses (E. I. Umnov, 1952) and in the task of the flow around the lattices of ellipses (D. A. Voytashevskiy, 1953). Series (3.8) and (3.9) are applied as analytical vehicle during the solution of direct and reverse/inverse problems in periodic latticed ranges and, in particular, for the numerical representation of this lattice onto the lattice of circles (L. A. Dorfman, 1952) and of the construction of lattice with distribution of the velocity, assigned on circumference in the equivalent lattice of circles (G. Yu. Stepanov, 1953; M. I. Joukowski, 1954).

Jet-edge tasks are simpler in that sense, that the jet boundaries become infinite, in other words, the infinite point lie/rests on boundary of the region, and all the jet streams are equivalent to each other and are reflect/represented to one and the same canonical range, for example, on the average without the center (see Fig. 2f).

The first generalization of Zhukovskiy - Chaplygin's jet-edge

task gave in 1934 N. I. Akhiyezer, who constructed the flow around the lattice of plates (according to the schematic of S. A. Chaplygin - A. L. Lavrentyev) with the descent of jets from trailing edge F_2 and certain point after entering edge; on the latter in this case, the velocity becomes infinite as during continuous flow. Then studied the flow around the final system of plates according to the same schematic (V. M. Abramov, 1936), lattices with the descent of jets at two points of plate (I. M. Belen'kiy and I. E. Zelenskiy, 1938), of lattice from the broken airfoil/profiles, consisting of the segments of two lines (N. V. Lambin, 1944). In all enumerated examples the solution easily is obtained according to the hodograph analysis of velocity whose range has so simple form that the composite potential in it is constructed directly or via conformal mapping from canonical range.

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The hodograph analysis of velocity will render/show the sufficiently effective resource the solution of the reverse problem, moreover not only for jet streams. The area of the hodograph of velocity is assigned sufficiently simple form, for example, by the limited circumference or the inversion of parabola (L. A. Simonov, 1940, 1941), by circular sector or the rounded off lune (G. Yu. Stepanov, 1949, 1962) or finally by the region of more general view which then

is reflect/represented cn the average with the aid of EGDA. In the latter case, it will be possible to acccmplish cf a series of requirements for the distribution of velocity cn airfoil/profile during design conditions cf flow (limitaticn of maximum speed and pressure gradients) and to construct lattices with practically important properties in the flow of the compressed viscous fluid (G. Yu. Stepanov, 1950, 1962).

The hodograph of the rate of jet stream is assigned in the form cf certain region (Fig. 8a). The characteristic parameters are the angles of entrance and output α_1 and α_2 and of velocity on back and the concave surface of airfoil/profile v^+ and v^-, constants on some sections. Then is manufactured the model of region in the form of electrolyyic bath, and according tc the already mentioned method of "division" is conducted its conformal mapping cn the average (Fig. 8b), moreover the length of cut/section with $v=v^-$ is selected from the condition of coinciding the critical pcint $d\phi/d\theta=0$, $\theta=\theta_0$) with point v=0. After obtaining of point conformity tc circumference and duct/contour of hodograph, the cascade profile (Fig. 8c) is constructed by the graphic or numerical integration:

$$z=\int_{\varphi_0}^{\varphi}\frac{d\varphi}{\bar{v}}=\int_{\theta_0}^{\theta}\frac{e^{i\alpha}}{v(\theta)}\frac{d\varphi}{d\theta}\,d\theta, \tag{3.10}$$

where the velocity potential on the circumference

$$\varphi(\theta) = \frac{\Gamma}{\pi}\frac{\theta}{2} - \frac{Q}{\pi}\ln\sin\frac{\theta}{2} \tag{3.11}$$
$$(\Gamma = v_1 t \sin\alpha_1,\quad Q = v_1 t\cos\alpha_1,\quad \theta_0 = \pi + 2\alpha_1).$$

As a result of construction, is known the conformity of the duct/contour of airfoil/profile and circumference s=s(θ); therefore the rate of flow v' under any off-design conditions is expressed by formula (3.3)

$$v'(\theta) = \frac{d\varphi'}{d\theta}\frac{d\theta}{ds} = \frac{\sin\left[\frac{1}{2}(\theta - \theta_0')\right]}{\sin\left[\frac{1}{2}(\theta - \theta_0)\right]}\frac{v_1'}{v_1}v(\theta).$$

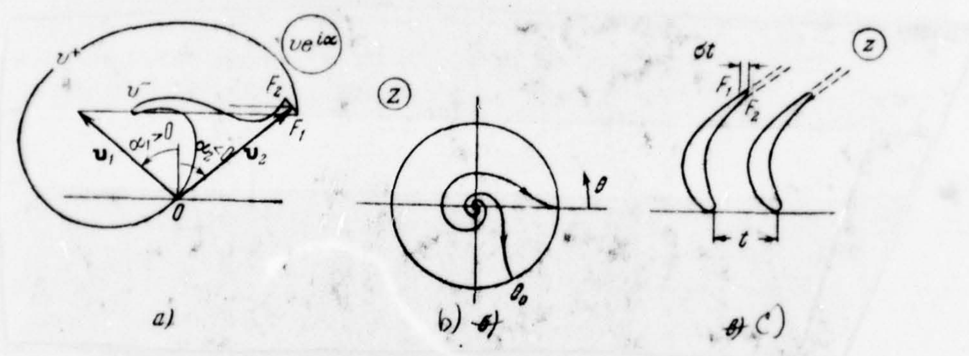

Fig. 8.

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According to described method were constructed several series of lattices over a wide range of a change in parameters $\alpha_1, \alpha_2, v^+, v^-$, satisfying the basic necessities of gas turbine engines [1].

FOOTNOTE [1]. It should be noted that the analogous methods shaped abroad were developed more lately (Zh. Revyuz, Rech. aeronaut., 1953, No. 31, 21-24; M. Khakeshmidt, ZAMM, 1961, 41: Sondern., 133-134). ENDFOOTNOTE.

The developed method was then generalized in the cases of motionless circular and double-row lattices (G. Yu. Stepanov, 1953, 1962).

In a number of the jet streams through lattices, they deserve reference several problems of the flow around bodies in channels with parallel walls. These problems as a result of the analytical continuation of the flow through the walls of channel give simultaneously the transverse flow around the corresponding lattice. The first of such solutions belongs to N. E. Joukowski, whom it examined in 1890 by its method the jet-edge flow around the wedge,

symmetrically arrange/located between parallel walls, which corresponds to the lattice of wedges (symmetrical broken airfoil/profiles). Were solved the analogous problems of the flat-plate flow and wedge according to the diagram of Efros with the recurrent stream, exiting to another sheet of flow plane (M. I. Gurevich, 1946, 1953), circle and ellipse (Ya. R. Bermann, 1949), some curvilinear arcs with the special distribution of speed (G. N. Pykhteyev, 1955).

From a series of special jet-edge diagrams and methods of their study, described in M. I. Gurevich's generalizing monograph (1961) and in his survey/coverage in present volume, to lattices were applied recently the mentioned diagram of Efros, - Zhukovskiy - Roshko's diagram with the output of jets to rigid direct/straight walls and diagram to Chaplygina - Yao -Tsu with the transition of jets to equidistant flow lines (A. G. Terent'yev, 1967). Last/latter diagram is obtained from S. A. Chaplygin's known diagram (1899) with the limited jet-edge zone in the vicinity of rear critical points on airfoil/profiles, if is not satisfied the condition of the closure/isolation of flow as a whole $(v_1 t \cos \alpha_1 < v_2 t \cos \alpha_2)$.

The most general solution of the problems of the jet-edge flow around the arbitrary systems of piecewise smooth duct/contours and lattices gave L. I. Sedov (1938, 1950), after modifying Joukowski's

method so that in plane parametric the variable u (see Fig. 2a) the problem it is reduced to the mixed boundary/edge whose solution is located from the functional equation of relatively unknown function $\alpha=\alpha(u)$ (angle of tangent inclination to duct/contour). The solution of this equation is simplified to quadratures, if streamlines consist of the segments of the lines (to which it corresponds α=const), and also for the inverse problem, in which is assigned function v (u), and α (u) is determined with an accuracy to some constants.

All the examined above methods of the solution of the problems of the cascade theories in one or the other form contained the solutions of linear boundary-value problems (Dirichlets, Neumann or mixed) for harmonic functions, in the majority of the cases of uniform or piecewise-uniform problems, and, as a rule, the selection of the unknown function, the form of canonical region and the methods of calculations specially they were not based. Between the fact precisely on this side of a question depend the success of the solution of problems and the effectiveness of results, that, in particular, most clearly showed the works of Moscow school in the problems of the cascade theories from fine/thin airfoil/profiles and jet streams.

Good results both in straight line and in reverse/inverse problems gives solution of heterogeneous boundary-value problem

relative to the function of Joukowski (or the logarithm of composite velocity) in half-plane, circumference or band.

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For the illustration of the developed methods, is given below the simplest example - the lattice of half-bodies - in physical plane and is examined function $\ln \bar{v}(\zeta) = \ln v - i\alpha$ in unit circle in plane Z (Fig. 8b) with the transition of the infinite points $z=\pm\infty$ respectively into points Z=0 and Z=1. If is known the conformity of the boundaries of airfoil/profile and circumference $Z = \exp i\theta,\ s = s(\theta)$, then according to Schwarz's integral in circle $|Z|<1$

$$\ln \frac{\bar{v}(\zeta)}{v_1} = \frac{1}{2\pi i} \int_0^{2\pi} \alpha(\vartheta) \frac{e^{i\vartheta}+Z}{e^{i\vartheta}-Z}\, d\vartheta \tag{3.12}$$

and, in particular, on the circumference

$$v(\theta) = v_1 \exp \frac{1}{2\pi} \int_0^{2\pi} [\alpha(\vartheta) - \alpha(\theta)] \operatorname{ctg} \frac{\theta-\vartheta}{2}\, d\vartheta; \tag{3.13}$$

it is analogous

$$\alpha(\theta) = \alpha_1 + \frac{1}{2\pi} \int_0^{2\pi} \ln \frac{v(\theta)}{v(\vartheta)} \operatorname{ctg} \frac{\theta-\vartheta}{2}\, d\vartheta. \tag{3.14}$$

Quadratures (3.13) and (3.14) are equivalent to the representations of functions ln v (θ) and α (θ) by the conjugated/combined Fourier series. In the center of the circle

$$v_1 = \exp \int_0^{2\pi} \ln v(\theta)\, d\theta, \tag{3.15}$$

$$\alpha_1 = \frac{1}{2\pi} \int_0^{2\pi} \alpha(\theta)\, d\theta. \tag{3.16}$$

Formulas (3.13)-(3.16) give the basis of solution of direct and reverse/inverse problems for the lattice of half-bodies.

In direct problem is known the function $\alpha(s)$ (with discontinuity/interruption on π at the critical point of smooth airfoil/profile). The angle of entrance α_1 and relative velocity on airfoil/profile are expressed respectively by integrals (3.13) and (3.16), in which function $\alpha(\theta) = \alpha(\theta(s))$ it is located by successive approximations, on the basis of the comparison of the potentials

$$\varphi(\theta) - \varphi(\theta_0) = \varphi(s), \tag{3.17}$$

where $\phi(\theta)$ is assigned by formula (3.11) and $\varphi(s) = \int_0^s v(s)\, ds$. The procedure indicated corresponds to the solution of nonlinear integrodifferential equation relatively α (6):

$$\frac{d\alpha}{d\theta} = \frac{1}{2} K(\alpha) \left[\operatorname{tg}\left(\frac{1}{2\pi} \int_0^{2\pi} \alpha(\theta)\, d\theta \right) - \operatorname{ctg}\frac{\theta}{2} \right) \times$$
$$\times \exp\left\{ \frac{1}{2\pi} \int_0^{2\pi} [\alpha(\vartheta) - \alpha(\theta)] \operatorname{ctg}\frac{\theta - \vartheta}{2}\, d\vartheta \right\}, \tag{3.18}$$

in which K (α) $= d\alpha/ds$ - curvature of the duct/contour of

airfoil/profile, α - the angle of tangent inclination (under integral - the angle of the slope of the velocity).

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This is equation analogous with the mentioned above functional equation jet-edge problem (L. I. Sedov, 1938, 1950) in the extreme case of degenerating the jets into the infinite point $z=+\infty$. An experiment in the calculations showed that the successive approximations converge well, and the described method can be considered most effective from known ones; it it is possible, in particular, to utilize for checking and refining different approximate solutions (L. A. Simonov, 1945, 1947; G. Yu. Stepanov, 1953, 1962).

Solution of the reverse problem during the distribution of velocity $v=v(s)$, assigned on airfoil/profile, is given by quadrature (3.14), in which function $v(\theta)=v(\theta(s))$ immediately it is located from equation (3.17).

Similar methods repeatedly were proposed and were applied both for the lattices and for single airfoil/profiles. Integrals of the type of of those entering formulas (3.13) and (3.14) were calculated by applying of quadrature formulas, harmonic analysis and different

adjoint functions (L. A. Simonov, 1945, 1950, 1957; Ya. M. Serebriyskiy, 1944; S.G. Nuzhin, 1947; G. Yu. Stepanov, 1962). Depending on the formulation of the problems, appear supplementary difficulties in connection with the determination of the permissible parameters of the problem. Thus, for instance, during the solution of the reverse problem the distribution of velocity and the flow parameters at infinity cannot be assigned arbitrarily, they must satisfy some supplementary conditions, equivalent to the conditions of closure/isolation and univalence of cascade profiles.

The special presentation of the theory of reverse/inverse boundary-value problems with numerous hydrodynamic application/appendices is in the monograph of S.G. Nuzhin and G. G. Tumasheva (1955, 1965). Inverse problems were solved for a single airfoil/profile (L. A. Simonov, 1945; G. G. Tumashev, 1946; G. I. Kostychev, 1958), for airfoil cascade during mapping onto ring (G. G. Tumashev, 1949), onto the lattice of plates without carrying out (L. A. Dorfman, 1952) and onto the lattice of circles (G. Yu. Stepanov, 1953; M. I. Joukowski, 1954), for a circular grating (V. V. Philipp, 1952; Yu. T. Borshchevskiy, 1959), the arbitrary final system of airfoil/profiles (R. M. Nasyrov, 1953), of three- and broadside arrays (A. M. Kazban, 1957, 1964), two- and triple lattices (F. M. Mukhametzyanov, 1964).

It is analogous as heterogeneous boundary/edge boundary-value problems, are solved questions concerning effect on the potential flow through the lattices of small stationary and unsteady deformations of airfoil/profiles, and also the motion of lattices, including rotations of circular gratings.

The calculation of effect on the flow of small deformations of the duct/contour of airfoil/profile and walls of narrow channel was one of the important computational application/appendices of variational methods of the theory of conformal mappings, developed by M. A. Lavrent'yev during the years 1930-1946 and used first of all to the problem of the existence of defined classes of wave and jet streams.

During, small stationary deformation of all airfoil/profiles in lattice, the conformal conformity of the boundaries of airfoil/profile and canonical region, necessary for calculating the integrals in formulas of type (3.12)-(3.16), is considered coinciding with this conformity for the undeformed airfoil/profile, and in higher example in question small displacement of the corresponding points on circumference $\Delta\theta$ is calculated by quadrature as function, conjugate/combined of normal (radial) deformation $\Delta\rho = \Delta n (d\theta/ds)$ (Δn - normal deformation of the duct/contour of airfoil/profile). Otherwise, it is possible to directly use formula (3.13) relative to small

increases Δv and Δα:

$$\Delta v(\theta) = \frac{v_1}{2\pi}\int_0^{2\pi}[\Delta\alpha(\vartheta) - \Delta\alpha(\theta)]\,\mathrm{ctg}\,\frac{\theta-\vartheta}{2}\,d\vartheta. \quad (3.19)$$

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Regarding the deformation of airfoil/profiles it is such, that the refinements of the conformity of boundaries is not required. However, this can be refined for the evaluation of the accuracy of result or during the solution of direct problem of the flow around the lattice, close to given one (theoretical) with known mapping onto canonical region which is taken as the first approximation.

The problem of the arbitrary unsteady deformation of airfoil/profiles or their motions during constant circulation in the potential flow is reduced to calculation by the quadratures of type (3.13) of supplementary tangent to duct/contour component/term v_s velocity on its given one normal component/term v_n or to the solution of the corresponding heterogeneous problem relative to the function of current or streaming potential of "displacement". The first task of this type - about the plane motion of liquid in the triangular cavity of the rotating body - was solved by N. E. Joukowski in 1885 (this task refers to flow in the rotating radial lattice with direct/straight blades). The rotation of single fine/thin airfoil/profile and two airfoil/profiles tandem was studied by L. I. Sedov in 1935; then it gave common/general/total approach to

the solution of similar problems within the framework of the theory of fine/thin airfoil/profile. The common/general/total properties of the flow through the rotating circular grating and, in particular, its conformal mapping onto straight line it examined P. A. Walter in 1926. The basic problems of the flow around such lattices are solved by G. I. Maykaparom(1949, 1953, 1958, 1966), by I. A. Dorfman (1956), by T. S. Solomakhovoy (1966). In relative motion through revolving gates, is obtained uniform task for a stationary flow with the constant eddying rot r = −2ω (ω=const - angular rate of rotation). The more complex problem of flow with the assigned (constant) eddying of the incident flow was solved for the single projection of flat/plane wall (S. A. Chaplygin, 1935), of limited Vikhrev regions in the potential flow (M. A. Lavrent'yev, 1959; M. A. Gol'dshtik, 1962), the transverse symmetrical flow around the lattice of circles with the "saw-tooth" airfoil/profile of inlet velocity (O. N. Ovchinnikov, 1961).

Produced survey/coverage makes it possible to assert that basic part of the contemporary hydrodynamic theory of lattices (the cascade theory from arbitrary airfoil/profiles in flat/plane potential incompressible flow) was created in our country. The solutions of the majority of problems were obtained more earlily or independent of foreign works, had clear practical directicality and important technical application/appendices.

To 1940 was created the cascade theory from fine/thin airfoil/profiles, were obtained the general ideas analytic functions in latticed region, are numerically solved by vortex/eddy method different problems, was constructed a series of new examples of jet streams, was used method of EGDA.

To 1950 is constructed and used in practical target/purposes the vast class of lattices from theoretical profiles, are obtained precise (in the form of a series) solutions of problems of the flow around the lattice of circles and lattice from arbitrary airfoil/profiles, are examined all the basic concepts of jet streams, is proposed the method of the shaping of lattices on the assigned hodograph of velocity with the application/use of EGDA for conformal mapping, are constructed the solutions of some inverse and direct/straight problems as boundary/edge in canonical regions. All these investigations were directed toward the satisfaction of the practical requirements of aviation and energy turbine construction.

During the subsequent years the methods indicated received further development and numerous practical application/uses. For the solution of direct problem, were utilized vortex/eddy method from ETsVM [ЭЦВМ - digital computer] and the method of conformal

mappings from EGDA.

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Inverse problem (shapings of lattices) was solved according to the hodograph analysis of velocity and as boundary/edge. Were examined also some special questions of the cascade theories. The results of theoretical studies and calculations widely were utilized in industry and they were generalized in special monographs (L. I. Sedov, 1939, 1950; N. Ye. Kochin, 1949; M. Ye. Deutsch and G. S. Samoylovich, 1959; M. I. Zhukovskiy, 1960; G. Yu. Stepanov, 1962).

§ 4. Lattice in plane flow of the compressed and viscous fluid.

Contemporary turbomachines usually work at the limited subsonic velocities of flow and with low-viscosity liquids; therefore, the compressibility effects and ductility/toughness/viscosity are small, although in practice precisely these effects determine the technical characteristics of turbomachines. In the cascade theory, as in airfoil theory, the account to compressibility and the viscosity of medium is conducted on the basis of the general methods of hydrodynamics, and also, this experimental investigations.

The common/general/total properties of the flat/plane

irrotational flow of the gas through the lattice depend on characteristic number $M = v/v_{зв}$ or given velocity $\lambda = v/v_{кр}$ (speed of sound $v_{зв} = \sqrt{\gamma p/\rho}$, critical speed $v_{кр} = \sqrt{2\gamma p^*/(\gamma+1)\rho^*}$, where p^* and ρ^* - pressure and density of the isentropically stagnation gas, γ - adiabatic index).

The equation of continuity (1.3) for the general case of the jet stream of gas takes the form

$$\rho_1 v_{1x} t = (1-\sigma)\rho_2 v_{2x} t, \tag{4.1}$$

where according to the equation of Bernoulli

$$\rho = \left(1-\frac{\gamma-1}{\gamma+1}\lambda^2\right)^{\frac{1}{\gamma-1}}\rho^*. \tag{4.2}$$

force $\bar{R} = R_x - iR_y$, which acts on the unit of the length of blade, is determined most simple from the theorem about momentum

$$\left.\begin{aligned} \rho_2 v_{2x}^2 (1-\sigma) t - \rho_1 v_{1x}^2 t &= (p_1 - p_2) t - R_x, \\ \rho_2 v_{2x} v_{2y} (1-\sigma) t - \rho_1 v_{1x} v_{1y} t &= -R_y. \end{aligned}\right\} \tag{4.3}$$

For continuous flow (σ=0) of expression R_x and R_y they are united and give a precise generalization of the N.E Joukowski theorum for a cascade profile in the flow of gas (L. I. Sedov, 1948; G. Yu. Stepanov, 1949); as a result of passage to the limit $t\to\infty$ when $\Gamma = \text{const}$ from this generalization it follows (for a single airfoil/profile)

$$\bar{R} = i\rho_\infty \bar{v}_\infty \Gamma. \tag{4.4}$$

(This propagation of Joukowski theorem in the case of the flow of gas

was for the first time proved for sufficiently small subsonic speeds in 1934 M. V. Keldysh and F. I. Frankl). Were proposed also different approximations of force R, acting on airfoil/profile in lattice, retaining form (4.4) (B. S. Stechkin, 1947; I. G. Loytsyanskiy, 1949; E. M. Berzon, 1949).

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The basic idea of theoretical studies of the flat/plane irrotational flow of gas, realized by S. A. Chaplygin in 1902, is passage from nonlinear equations of motion in flow plane

$$\left.\begin{aligned} v_x &\equiv v\cos\alpha \equiv \frac{\partial\varphi}{\partial x} = \frac{\rho^*}{\rho}\frac{\partial\psi}{\partial y}, \\ v_y &\equiv v\sin\alpha \equiv \frac{\partial\varphi}{\partial y} = -\frac{\rho^*}{\rho}\frac{\partial\psi}{\partial x} \end{aligned}\right\} \quad (4.5)$$

to linear equations in the hodograph plane of speed. The latter in the form of I. S. Leybenzon (1935) take the form

$$\frac{\partial\varphi}{\partial\alpha} = \sqrt{K}\frac{\partial\psi}{\partial s}, \quad \frac{\partial\varphi}{\partial s} = -\sqrt{K}\frac{\partial\psi}{\partial\alpha}, \quad (4.6)$$

where $K=(\rho^*/\rho)^2(1-M^2)$ - the function of S. A. Chaplygin and $ds=\sqrt{1-M^2}dv/v$. In connection with the fact that equations (4.5) and (4.6) as a result of passage to the limit with $M\to 0$ are converted into the equations of motion of the incompressible fluid, but the compressibility effect of gas on all flow parameters at small subsonic speeds is small (it is of the order M^2), in the majority of theoretical studies are utilized the corresponding flows of the

incompressible fluid around constart/invariable (as the first approximation) or deformed airfoil/profiles (P. A. Val'ter, 1932-1936; N. A. Slezkin, 1934; S. A. Khristianovich, 1940; L. I. Sedov, 1946-1949).

In practice for a single airfoil/profile and then for airfoil cascade were utilized different approximation formulas of form $v = v(V, M_\infty)$ for the "recalculation" of speeds V of the incompressible fluid around the same airfoil/profile (S. A. Khristianovich and Ya. M. Serebriyskiy, 1942, 1944; V. S. Polyadskiy, 1943; S.G Nuzhin, 1946; G. F. Burago, 1950, etc.). For airfoil cascade, this method is generally little suitable. At low speeds for the fine/thin weakly bent airfoil/profiles, is most substantiated the simplest correction of Prandtl - Glauyerb $v = V/\sqrt{1-M_\infty^2}$. Accordingly entire/all the cascade theory of a small denseness from fine/thin airfoil/profiles is directly spread to the case of the flow of gas by the affine transformation of the flow plane of the incompressible fluid - elongation in relation $1/\sqrt{1-M_\infty^2}$ in the direction, perpendicular to the average speed of flow ($v_\infty = {}^1/_2 (v_1 + v_2)$). It is important that in this case change the space and the angle of forward projection of lattice.

For airfoil cascades of arbitrary form, is applied the approximation method of S. A. Chaplygin (1902), according to whom real dependence (4.2) of gas density on speed is replaced by that

approximated

$$\rho=\frac{\rho^*}{\sqrt{1+4C^2\lambda^2}},\qquad (4.7)$$

with which K=1 and equations in hodograph plane in accuracy transfer/convert into the equations of Caucky - Riemann. In formula (4.7) is one essential the arbitrary constant C. When selecting $C^2=1/[2(\gamma+1)]$ real dependence (4.2) is approximated with an accuracy down to the terms of order $C^4\lambda^4$; if $C^2=0.3$, then an error in approximate dependence (4.7) with $\lambda<0.95$ does not exceed 2o/o. To the flow of S. A. Chaplygin's gas in plane z with a speed of $\lambda e^{i\alpha}$ it is possible to supply in conformity certain fictitious flow of the incompressible fluid in plane ζ with speed $\Lambda e^{i\alpha}$ ($\lambda=v/v_{\text{кр}}$, $\Lambda=V/v_{\text{кр}}$), moreover

$$\left.\begin{aligned}\lambda&=\frac{\Lambda}{1-C^2\Lambda^2},\\ dz&=d\zeta-C^2(\Lambda e^{i\alpha})^2\,d\bar{\zeta}.\end{aligned}\right\}\qquad (4.8)$$

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For Chaplygin's gas, adiabatic curve $p\rho^{-\gamma}=\text{const}$ is replaced by straight line in coordinates (p, ρ^{-1}) (therefore, method calls also the method of the linear approximation of adiabatic curve), and Zhukovskiy theorem for a lattice is proved to be valid in its usual form (4.4), if is is counted $\rho^{-1}=1/2(\rho_1^{-1}+\rho_2^{-1})$ (L. G. Loytsyanskiy, 1949).

The theory of the flow around airfoil/profile in this

approach/approximation for jet-edge and continuous noncirculatory flow was developed by N. A. Slezkin (1935, 1937), also, abroad K. Jakob (C. r. Acad. Sci., 1936, 203:7, 423-425), S. Tsyanem (J. Aeronaut. Sci., 1939, 6:10, 399-407) et al.; circulation flows examined in 1940 of S. A. Khristianovich and in 1946. L. I. Sedov, Ts Lin' (Quart. Appl. Math., 1946, 4:3, 291-297) and P. Zhermen (C. r. Acad. Sci., 1946, 223:15, 532-534).

As a result of developing this theory, all methods of the solution of the problems of the flow around hydrodynamic lattices of the incompressible fluid were generalized, in Chaplygin's approximate setting, for the case of the subsonic flow of gas.

First of all, to any jet stream (or the flow through the lattice of half-bodies), examine/considered in plane ζ, according to relationship/ratios (4.8) it answers the flow of the gas through the deformed lattice, which has another period

$$T_1 = T - C^2 (\Lambda_1 e^{i\alpha_1})^2 \bar{T}, \qquad (4.9)$$

another thickness of jets at infinity and another rate of flow (G. Yu. Stepanov, 1949; G. A. Bugaenko, 1949).

During, the construction of lattice on the assigned hodograph of speed, is assigned the range of the hodograph of the fictitious flow of incompressible fluid $\bar{V} = Ve^{-i\alpha}$, in it is determined the composite

potential $w=w(\bar{V})$, after which the boundary of the flow of gas they are located by contour integration of the hodograph:

$$dz = e^{i\alpha}\frac{d\varphi}{v} = e^{i\alpha}(1 - C^2\Lambda^2)\frac{d\varphi}{V}. \qquad (4.10)$$

In the more complex case of continuous flow, composite potential at the terminuses of vectors $\bar{V}_1$ and $\bar{V}_2$ has logarithmic special feature/peculiarities (sources of vortex into vortex drain) with the intensities

$$\Gamma_{1,2} + iQ_{1,2} = \pm\left(\sin\alpha_{1,2} + i\frac{\rho_{1,2}}{\rho^*}\cos\alpha_{1,2}\right)v_{1,2}t. \qquad (4.11)$$

For obtaining the physically real flows, it is necessary to ensure coincidence critical of points $dw/d\bar{V}=0$ with point $V=0$, which is reached via the appropriate selection of the parameters of flow or form of the duct/contour of hodograph. Thus, the described in § 3 method of the shaping of lattices is spread to the flow of gas, moreover, under the design conditions of flow is provided the limitation of velocities on the airfoil/profile of subsonic $\Lambda < \Lambda_{кр} = 2/(1+\sqrt{1+4C^2})$ (G. Yu. Stepanov, 1949, 1962).

In the case of the continuous circulation flow through the assigned lattice, the passage from the flow of the incompressible fluid to the flow of gas becomes complicated by the fact that in this case according to formula (4.7) in the plane of gas the flow is not single-sheet, with different periods T_1 and T_2 at infinity before the lattice and after it, moreover to each circuit/bypass of

airfoil/profile corresponds displacement to the vector

$$\tau = \oint dz = T_2 - T_1 = C^2[(\Lambda_1 e^{i\alpha_1})^2 - (\Lambda_2 e^{i\alpha_2})^2]\bar{T} + \frac{2C^2}{v_{\text{кр}}^2}\Gamma V_\infty e^{i\alpha_\infty}. \quad (4.12)$$

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This difficulty, identical for a single airfoil/profile and for a lattice, was overcome in the fortieth years by several authors, who proposed different reception/procedures of the provision for a closure/isolation of airfoil/profiles in the flow of gas. All these reception/procedures, actually, are connected with obtaining in the plane of the fictitious incompressible flow of the airfoil/profiles, open-circuited to vector $(-\tau)$.

L. I. Sedov in 1946, supplemented function $\zeta=\zeta(Z)$, the representing interior of circle from plane parametric alternating/variable Z (Fig. 7c) to the exterior of theoretical profile (and of theoretical airfoil cascade) by the members of form $-(\tau/2\pi i)\ln(Z - Z_\infty)$, because of which in plane z of the flow of gas are obtained the locked airfoil/profiles, close to initial theoretical airfoil/profiles in the incompressible fluid. Abroad the analogous reception/procedure of the construction of lattices in the flow of gas was used more lately Ts. Linem (J. Math. and Phys., 1949, 28:2, 117-130). Examples of the construction of lattices from theoretical profiles (of Chaplygin's type) it gave A. I. Burimovich in 1950.

With S. A. Khristiancvich and I. M. Yur'yev, developing in 1947 the first approximation of the method of Khristiyanovich (actually coinciding with the approximation method of S. A. Chaplygin), they ensured the closure/isolation of theoretical profiles, after changing circulation in the range of hodograph. In this case, critical points cease to coincide with point $\overline{V}=0$ (speed in them non-vanishing) and in their vicinity are obtained the characteristic infinite "whiskers", which exit to the second sheet of plane z. For a lattice this method indicates a circulation control of source of vertex and vortex discharge according to formula (4.11).

R. M. Fedorov in 1951 on B. S. Stechkin's proposition used the conformal conversion of the range of hodograph by function $V_* = V/(1 - C^2\overline{\Lambda}^2)$, moreover, with an accuracy to small ones order $C^4\Lambda^4$ are obtained the locked airfoil/profiles both in the plane z, and in the plane of auxiliary incompressible flow $\zeta_* = dw/d\overline{V}_*$, and is accurate transfer equation

$$dz = d\zeta_* + C^2\Lambda^2 (e^{2i\alpha} d\overline{\zeta}_* + e^{-2i\alpha} d\zeta_*).$$

All the described above reception/procedures of the construction of theoretical lattices were studied and compared based on one example airfoil cascade, used in turbines (G. Yu. Stepanov, 1954, 1962).

It is simple, actually by applying formulas (4.7)-(4.10), are solved the reverse/inverse problems of the construction of lattices in the flow of gas with the specified distribution of speed on the duct/contour of airfoil/profile or canonical range; the majority of the methods, described into § 3, was common for the flow of Chaplygin's gas, moreover the corresponding boundary-value problems were solved for the speed of fictitious incompressible flow and were made the condition of the closure/isolation of airfoil/profiles in the flow of gas (G. G. Tumashev, 1945, 1949, 1965; V. V. Philipp, 1952; G. Yu. Stepanov, 1953, 1962; M. I. Joukowski, 1954, 1956; L. A. Dorfman, 1954, 1955). Abroad analogous methods were developed by J. Lighthill (ARC Repts and Memor., 1945, No. 2104), by J. ~~K~~Lostello (NACA Rept, 1950, No. 978), J. D. Stanitsem (NACA Rept, 1953, No. 1115) et al.

More complex direct problem of the flow around lattice from the assigned airfoil/profiles is reduced to the solution of the functional equation, analogous to equation (3.18), moreover is considered communication/connection between planes z and ζ, and also by speeds v and V (4.8); a good first approximation it gives the solution of this problem for the incompressible fluid.

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The corresponding equations for the jet-edge and noncirculating flow around single airfoil/profiles were shown by N. A. Slezkin in 1934, and for circulation flow around single airfoil/profile and airfoil cascade - L. I. Sedov in 1946. Examples of the flow-field analysis of the assigned lattices were carried out more lately (Ya. M. Kotlyar, 1954, 1955; G. Yu. Stepanov, 1953, 1962). For the simplest example, of the flow around the lattice of the wedges, which schematize angular entering edges, solution it is obtained in explicit form and gives the expression of calculated angle of attack

$$\alpha_1 - \alpha_{cp} = \frac{\gamma}{\pi} \operatorname{arctg} \left(\frac{\rho^*}{\rho_1} \operatorname{tg} \alpha_1 \right).$$

According to this expression the calculated flow angle α_1, measured from standard to the front of lattice, exceeds the mean angle of edges α_{cp}, where the difference in these angles grow/rising with the increase of apex angle γ and of Mach number (G. Yu. Stepanov, 1962).

The basic results of the cascade theories in the subsonic flow of gas were obtained in Chaplygin's approximate setting with K=const. As Yu. V. Rudnev in 1949, it generalized Chaplygin's precise method to the case of arbitrary dependence $\rho=\rho(p)$ and such flows whose composite potential has special feature/peculiarities within the

range of hodograph, after examining as an example the jet-edge flow around the lattice of plates. G. A. Dabrowski in 1950 developed the method, instituted on the approximation of higher order, form $K = C_1 \,\mathrm{th}^4 C_2 s$ (C_1, C_2 - arbitrary constants), and he solved by this method a large number of different problems, including jet-edge flow around the lattice of plates (1955, 1964).

For the lattices of large denseness practical application/use obtained some approximate methods of the calculation of flow in channels, instituted mainly on different assumptions about form and curvature of equipotential lines and flow lines (G. Yu. Stepanov, 1953, 1958, 1962; G. S. Samoylovich, 1954, 1959; M. I. Joukowski, 1960). In combination with the more precise flow-field analyses the input and trailing edges of airfoil/profiles these methods it became as a whole satisfactory approximate solution of direct problem, in principle suitable for any transonic and supersonic speeds (in the absence of shock waves).

The transonic and mixed cascade flows specially were not studied. One should note approximate solution of YU. V. Rudneva (1952), relating to the mixed flow through the symmetrical lattice from rhombs, and also the cycle of the works of F. I. Frankl and his colleagues with transonic flows in the channels (see the survey of R. G. Barantseva, 1965).

The supersonic flows through lattices were examined in linear approach/approximation in connection with wings with internal framework (V. V. Keldysh, 1957), to airfoil/profiles in the squeezed flow (M. D. Khaskind and V. S. Khomenko, 1958), to lattice in conical flow (N. V. Smirnov, 1961, 1962, 1966). Interest in nonlinear problems is connected with the problem of supersonic compressor and by the so-called expansion in skew shear of the vane channels of the lattices of turbines. The known possibility of the construction of complex supersonic flows via the "cementing" simpler was used for the construction of lattices with the channels of constant width (M. F. Zhukov, 1950) and of turbine cascades behind the shortest nozzles with points of inflection (G. Yu. Stepanov, 1953, 1962). For the observing with supercritical jump/drops pressure deviation gas in skew shear of turbine cascades were proposed different approximation and semi-empirical formulas.

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Most widely known of them follows from the one-dimensional equation of the continuity

$$\cos\alpha_2 = \frac{1}{\sigma q_2}\frac{a}{t}, \qquad (4.13)$$

where a - width of the narrow section of channel, $\sigma = p_2^*/p_a^*, p_a^*$ - the

total pressure in section a, $q_2 = (\rho_2 v_2)/(\rho_{2кр} v_{кр})$, and value σ is selected empirically.

By applying the one-dimensional equations of conservation for the angle of deflection of nonviscous gas after the lattice of plates obtained a precise for this model formula (G. Yu. Stepanov, 1952)

$$\operatorname{tg}\delta = \frac{1}{\gamma+1-p}\left[\sqrt{\left(\frac{\gamma}{\gamma-1}p\operatorname{tg}\alpha_к\right)^2+\frac{\gamma+1}{\gamma-1}(1-p)^2}-\frac{\gamma}{\gamma-1}p\operatorname{tg}\alpha_к\right], \quad (4.14)$$

where $\delta = \alpha_к - \alpha_2$, $\alpha_к$ - the angle of setting plates, $p = p_2/p_{кр}$, $p_{кр}$ - critical pressure in narrow section. At the obtained angle of departure α_2, value σ is located from equation (4.13); condition $\sigma = \sigma_{min}$ determines the limit of expansion in skew shear. Formula (4.14) was common for the cases of the edges of final thickness with evacuation/rarefaction after them, the presence of friction in skew shear and of indirect edges (G. Yu. Stepanov, 1953, 1962; V. L. Epstein, 1961; V. V. Goltzes, 1959, 1963).

In the fiftieth years was expand/developed the discussion on the so-called "directing" action of infinite lattice, exerted by it to the incident supersonic flow. This discussion was instituted on that obvious theoretical conclusion/derivation that the supersonic flow before the lattice of plates (when $v_1 \cos\alpha_1 \leqslant v_{зв}$) can have the only direction, which coincides with the direction of plates. However, this conclusion contradicted the experimental data, obtained in

turbomachines. Contradiction was explained by the emergence of breakaway on entering edges and by the corresponding rearrangement of the supersonic flow. The same considerations which were used for the derivation of formula (4.14), made it possible to calculate the parameters of the flow, which was flushed between plates after breakaway (G. I. Taganov, 1952). At low speeds is obtained known formula for the factor of loss of kinetic energy of flow, which flows around the lattice of plates with angle of attack δ:

$$\zeta_{\text{вх}} = \frac{p_1^* - p_\text{к}^*}{\frac{1}{2}\rho_1 v_1^2} = \frac{\sin^2\delta}{\cos^2\alpha_\text{к}}. \quad (4.15)$$

Abroad the developed theory of entry loss into lattice appeared on several years later [1].

FOOTNOTE [1]. ~~Dzh~~ J. J. Cramer and J. D. Stanitz, NACA Techn. Note, 1955, No. 3149; M. Lyudevig, Forsch. Geb. Ingenieurwesens, 1956, 22 : 6, 181—191. ENDFOOTNOTE.

Were analogously defined the parameters of the flow, passing through the perforated/punched wall, considered as lattice of cuts of one to straight line (G. L. Grodzovskiy, 1953), and also loss during the flow through circular gratings - motionless and rotating (V. T. Mitrokhin, 1958, 1960; N. B. Trainer, 1967).

Many authors discussed the possibilities of the electrical simulation of the flows of gas, especially advisable for equations

form (4.6) in the area of the hodograph of speed. Examples of the realization of this analogy in the bath of alternating/variable depth (by inversely proportional $\sqrt{K}$), carried out in 1949 a. M. by Luxemburg, they showed, that under actual conditions equipment errors in the simulation not lower than fundamental errors in the approximation method of Chaplygin.

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Therefore practical application/use simulation was obtained only for conformal mapping of the area of the hodograph of speed V of the fictitious flow on the average (G. Yu. Stepanov, 1950, 1962).

The interesting possibilities of qualitative investigation, especially for supersonic speeds, afforded the gas-hydraulic analogy between the motion of gas and gravity solution with floating surface. This analogy, indicated by N. Ye. Joukowski in 1912, was applied for the demonstration of different supersonic flows, including through the step/stage of turbine during relative motion of lattices (G. F. Kamnev, 1957).

The viscosity of liquid in plane flow through the lattice is exhibited in formation/education on the airfoil/profiles of thin boundary layer, in flow breakaway, at least at trailing edge, the

appearance after it of evacuation/rarefaction and the equalization of traces after lattice. In this case, appear, usually small, changes of the parameters of the main flow in comparison with those determined in the ideal model of nonviscous liquid. These changes in the one-dimensional examination of flow are characterized by the estimated factors of loss ζ, of the velocity (momentum/impulse/pulse) ϕ, expenditure/consumption μ and energy η, which enter in the hydrodynamic computation of turbomachine.

In the uniform flow

$$\varphi = \frac{v}{v_0}, \quad \mu = \frac{\rho v}{\rho_0 v_0}, \quad \eta = \frac{\rho v^2}{\rho_0 v_0^2} = \varphi\mu = 1 - \zeta \qquad (4.16)$$

(index of "0" is related to the parameters of ideal flow in the section in question). In the nonuniform flow of communication/connection between estimated coefficients, they depend even on the characteristic of the nonuniformity of the field of velocities. The enumerated coefficients together with angle of departure α_2 determine basic lattice parameters and turbomachine; their measurement is the basic goal of the experimental studies of lattices.

In the theory of the flow of the viscous fluid through lattices, enters the boundary-layer calculation on airfoil/profile, the account of the thickness of trailing edges and flattening of flux after

lattice. The first calculations and the measurements of boundary layer on cascade profiles pertain to the year 1946 and belong to A. S. Zil'berman and N. M. Markov. L. G. Loytsyanskiy generalized the known method of the approximate computation of the profile drag of wing to the case of lattice and expressed the factor of loss ζ through thicknesses δ_{κ}^{**} of the loss of momentum/impulse/pulse in boundary layer on trailing edges (in 1947 for the incompressible fluid and in 1949 for gas); N. M. Markov in 1947 proposed the expression of coefficient ζ through thicknesses δ_{κ}^{***} of energy loss. In the case of lattice, however, unlike single airfoil/profile, it proved to be possible with the aid of only equations of conservation to more strictly solve this problem and to express by the known parameters of boundary layer in the plane of the trailing edges (is below index "k") all parameters of the flushed flow after lattice (G. Yu. Stepanov, 1949, 1962):

$$\left.\begin{aligned} \eta \equiv 1-\zeta &= \frac{\mu_{\kappa}^2 \cos^2\alpha_{\kappa} + \varphi_{\kappa}^2 \sin^2\alpha_{\kappa}}{1-\Delta\bar{p}}, \\ \operatorname{tg}\alpha_2 = \frac{\varphi_{\kappa}}{\mu_{\kappa}}\operatorname{tg}\alpha_{\kappa},\quad \Delta\bar{p} &= \frac{p_2-p_{\kappa}}{\frac{1}{2}\rho_{0\kappa}v_{0\kappa}^2} = 2\mu_{\kappa}(\varphi_{\kappa}-\mu_{\kappa})\cos^2\alpha_{\kappa} + \bar{p}_{\kappa p}\sigma. \end{aligned}\right\} \quad (4.17)$$

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In formulas (4.17) $\mu_{\kappa} = 1-\sigma-\delta_{\kappa}^{*}/(t\cos\alpha_{\kappa})$, $\varphi_{\kappa} = 1-\delta_{\kappa}^{**}/(\mu_{\kappa}t\cos\alpha_{\kappa})$, $\sigma = d/(t\cos\alpha_{\kappa})$ - the dimensionless thickness of separation zone after edge (in the direction of the front of lattice), $\bar{p}_{\kappa p} = 2(p_{\kappa}-p_{\kappa p})/(\rho_{0\kappa}v_{0\kappa}^2) \approx -0{,}10$ - dimensionless pressure after edges, δ_{κ}^{*} - the sum of the displacement

thicknesses of boundary layer from two sides of trailing edge, δ_{κ}^{**} - the same in the relation to the momentum thickness. Analogous formulas were obtained abroad later and only in the particular case of the incompressible fluid and infinitely fine/thin trailing edges (G. Schlichting and N. Shol'ts, Ingr-Arch., 1951, 19: 1).

For the calculation of the development of trace after lattice, was used the theory of free turbulent jet, which made it possible to rate/estimate the nonuniformity of flow depending on distance after edges and the corresponding average parameters of flow (G. Yu. Stepanov, 1953, 1962; Ye. V. Solokhin, 1957).

The loss factor for tentative calculations is expedient to break on two parts: $\zeta = \zeta_{\text{кр}} + \zeta_{\text{тр}}$. The first part, the loss factor due to the final thickness of trailing edges, is determined by semi-empirical formulas, for example, $\zeta_{\text{кр}} = -\bar{p}_{\text{кр}}\sigma$ or $\zeta_{\text{кр}} = 0{,}18\ d/a$, ~~a~~ and the second depends only on friction on airfoil/profile, i.e., on the parameters of boundary layer. With completely turbulent layer in the incompressible fluid, is valid the estimated formula

$$\zeta_{\text{тр}} = \frac{C}{\text{Re}^m}\left(\frac{s}{t}\right)^{1-m}\left[\left(\frac{v^+}{v_2}\right)^n+\left(\frac{v^-}{v_2}\right)^n\right], \qquad (4.18)$$

in which s - length of the camber line, v^+ and v^- - some average speeds, respectively on back and the concave surface of airfoil/profile, $\text{Re} = v_2 t/\gamma$. In the simplest case of boundary-layer

calculation with $Re=10^5-10^7$ (L. G. Loytsyanskiy, 1941, 1957) C=0.072, m=0.20 and n=3.1. With the developed roughness of the surface of airfoil/profile, is valid the same formula, in which instead of Re number project/emerges the relative smoothness of surface t/Δ (Δ - average quadratic height of irregularities), and it is possible to accept C=0.048, m=0.25, n=2 (K. K. Fedyaevskiy and N. N. Fomin, 1936, 1939). On the more contemporary data C=0.032, m=0.20 and n=2.25.

For the cooled turbine blades within the limits of the validity of Reynolds's hypothesis about the similarity of dynamic and thermal boundary layers dimensionless heat-transfer coefficient into blade $q = Q/[c_p G (T_4^* - T_{\text{ст}})]$ (Q - heat flow, c_p - the heat capacity of gas at a constant pressure, G - gas flow, $T_{\text{ст}}$ - the temperature of wall) can be rate/estimated on the same formula (4.18) with C=0.024, m=0.167, n=0.833 (L. M. Zysina-Molozhe, 1957; G. Yu. Stepanov and V. L. Epstein, 1958, 1962).

The numerous estimated and more precise boundary-layer calculations and profile losses under varied conditions showed, on the whole, satisfactory coincidence with experimental data during nonseparated flow (I. L. Povkh, 1950; N. M. Markov, 1952, 1955; A. S. Ginevskiy, 1954; L. M. Zysina-Molozhe, 1955). One should note application/use in these calculations of ETsVM (V. M. Zelenin and V. A. Shilov, 1964). On the basis of the obtained dependences, were

conducted the generalizations of experimental data and the perfection/improvement of the procedure of the experimental studies of foil lattices.

At the off-design angles of the entrance when the flow around lattices occurs with flow breakaway on airfoil/profile, won acceptance semi-empirical formulas; most substantiated of them take the following form:

$$\zeta = A + B\left(\frac{\cos\alpha_2}{\cos\alpha_1}\right)^2 + C\left[\frac{\sin(\alpha_1 - \alpha_{1\,\text{расч}})\cos\alpha_2}{\cos\alpha_1\cos\alpha_{1\,\text{расч}}}\right]^2, \qquad (4.19)$$

where $A + B(\cos\alpha_2/\cos\alpha_{1\,\text{расч}})^2 = \zeta_{\text{расч}}$, A=(0.4-0.6) B and C=0,1-0,3 (G. Yu. Stepanov, 1958); B=0,058, C=0,265 (V. I. Epstein, 1959).

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The effects of the flow of real liquid include additional losses in two-phase flow and precipitation of the second phase to blades. These effects, and also the motion of liquid film along the surface of blades were studied in semi-empirical treatment conformably mainly to the flow of wet steam in turbines (M. D. Vaysman, 1953, 1967; R. M. Yablonik, 1960; E. I. Marchik and V. L. Epstein, 1960, 1965).

For the theoretical shaping of lattices in the flow of viscous fluid, were utilized reverse/inverse methods and, especially, the

hodograph analysis of the speeds which provide hydrodynamically the advisable distribution of the speed of nonviscous liquid - with the limited maximum velocities (v^{+}), also, with a minimum number of diffuser sections ($dp/ds<0$) with the guaranteed nonseparated flow, for which must be $(\delta^{**}/\rho)\,(dp/ds) = \xi < \xi_{\text{кр}}$. It was suggested to shape these sections with equiprotable breakaway ($\xi = \text{const} < \xi_{\text{кр}}$); they were placed and solved the simplest problems of the construction of optimum lattices with the smallest profile losses which confirmed a series of the empirical rules of the construction of airfoil/profiles (G. Yu. Stepanov, 1950, 1958, 1962; M. E. Deutsch and G. S. Samoylovich, 1959; M. I. Joukowski, 1960; E. A. Gukasova et al., 1960).

As has already been indicated, large practical role they play the experimental studies of lattices. Apparently, the first in the world tests with the models of foil lattices ran in 1902 N. E. Joukowski in the wind tunnel of Moscow University. It is simultaneous with the development of the cascade theories, the successes of construction and calculation of turbomachines, and also with the increase of the technique of aerodynamic experiment were developed and were improved the methods of the experimental study of lattices and grow/rose the practical role of their results. In the fiftieth years in our country, were created, in essence, in scientific research institutes and at the plants of the aircraft industry and turbine

construction, the original construction/designs of the installations, during which were carried out the important studies of lattices, which had high value for aviation and rocket engineering and for power engineering. The part of these installations and investigations is described in the vast literature the examination by which exceeds the scope of this survey.

The developed methods the account of the compressibility effects and viscosity of liquid, and also the results of experimental investigations completed the development of the examined above cascade theory in flat/plane steady flow and were utilized in practical application/appendices.

§5. Unsteady flow around lattices.

The flow around lattices in turbomachines is unsteady in essence because of relative motion of the rotating and stationary parts, oscillation/vibrations of elastic blades and disks, and also in communication/connection with separation phenomena and the oscillation/vibrations of flow as a whole. The examined above steady flows are the simplified model of the steady on the average flow, a special case with respect to overall unsteady motion. The difficult problem of the unsteady flow around lattices attracts in recent years continually increasing attention, since unsteady aeroelastic

phenomena increasingly more frequently prove to be the main reason, which determines the reliability of turbomachines and which limits their highest efficiency or the smallest weight.

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The problem of the aeroelasticity of blades, housings and disks of turbomachines as a whole is still distant from complete theoretical completion; the obtained solutions of separate particular problems rest on ideas and the methods of unsteady aero- and hydrodynamics, developed in the first half of our century in connection with the analogous problem of the unsteady flow of the wing or aircraft, and on experimental investigations.

The contemporary theory of the unsteady flow of wings and lattices utilizes the simplified vortex/eddy models of the flow of nonviscous liquid - actually, N. E. Joukowski's idea about connected and free vortices.

The first fundamental result in this area belongs to S. A. Chaplygin, who gave in 1926 the common/general/total expressions of main vector and main moment of the unsteady forces, which act on solid body in flat/plane incompressible flow during constant circulation. L. I. Sedov obtained in 1935 analogous expressions in

the more general case of the arbitrarily variable duct/contour and during alternating/variable circulation. During the years 1924-1929 in the works of L. Prandtl, V. Birnabaum, G. Wagner and G. Glauyerb were developed the approximate vortex conceptions of unsteady motion of fine/thin wing with the line of discontinuity of speed, the vortex wake, appearing in connection with alternating/variable circulation around airfoil/profile during satisfaction of Zhukovskiy - Chaplygin's condition on trailing edge. The effective, in the form of final formulas, calculation of forces during small harmonic oscillations of wing was produced in 1935 by M. A. Lavrent'yev and M. V. Keldysh and another method by L. I. Sedov. A. I. Nekrasov in the monograph, which pertains to the year 1941 (published in 1947), gave detailed survey/coverage and generalization of airfoil theories in the unsteady flow from Soviet and foreign works, including the cases of the finite-span wing (N. Ye. Kochin, 1941), of the heterogeneous incident flow, of aperiodic motion and compressed medium. In 1947 were published the new and more advanced solutions of the problems of the oscillation of fine/thin wing at subsonic (M. D. Khaskind) and supersonic speeds (Ye. A. Krasilsyuchyuikov, I. A. Panichkin, M. D. Khaskind and S. V. Fal'kovich, L. A. Galinas, M. I. Gurevich), including for the first time for the finite-span wings different planform. The systematic presentation of linear airfoil theory in supersonic unsteady flow is in Ye. A. Krasil'shchikovoy's monograph (1952).

Deserves special reference created in of the first of the enumerated above works fortieth years the theory of the formation/education of the thrust/rod of the oscillating wing (because of suction force), supplemented then by the calculation efficiency of wing (M. D. Khaskind, 1944) and of the thrust/rod of the waving wing in connection with the formation/education after it of path/track from discrete eddy/vortices (V. V. Golubev, 1944-1946, 1957).

The first results in the unsteady cascade theory belong to L. I. Sedov (1939, 1950), who indicated the general solution of the problem of the arbitrary synchronous motion of the airfoil/profiles of foil lattice in irrotational incompressible flow, in a precise setting during constant circulation and in quasi-stationary (without free vortices) - during alternating/variable circulation. In particular, it gave the common/general/total expressions of apparent additional masses λ (β) airfoil/profile in lattice gave the results of their calculation for the lattice of plates with arbitrary carrying out. The simplest form these expressions have in the cases of lattices without carrying out (β=0), also, from cuts of one a straight line ($\beta=1/2\pi$):

$$\lambda(0)=\frac{2}{\pi}\rho l^2 \ln \operatorname{ch}\frac{\pi t}{2l}, \quad \lambda\left(\frac{\pi}{2}\right)=-\frac{2}{\pi}\rho l^2 \ln \cos\frac{\pi t}{2l}. \qquad (5.1)$$

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Were more lately calculated the apparent additional masses of lattice from rectangles and the double-row lattice of plates (M. I. Gurevich, 1940, 1954). Analogously in common/general/total setting is solved the problem of the shock of lattice during continuous and jet-edge flow; so was examined the shock of the lattice of plates during their symmetrical cavity flow according to the diagram of D. A. Efros (S. I. Parkhomovskiy, 1958). It should be noted that the knowledge of apparent additional masses makes it possible to simply solve the acoustic problem of the reflection of sound by sufficiently frequent lattice and to obtain G. D. Malyuzhints's known formula for the coefficient of the sound conduction of lattice (M. I. Gurevich, 1964).

On the basis of the developed overall theory, all methods of the solution of the problems of irrotational flow of the incompressible fluid through lattices elementary are generalized to the case of the arbitrary identical motion of their airfoil/profiles in the irrotational flow. In this case, instead of the motionless or stationary driving/moving lattice, is examined the lattice from given one on airfoil/profile in the function of time τ by normal speed

$v_n = \partial\varphi/\partial n = v_n(s, \tau)$ or by the function of current $\psi = \int_0^s v_n(s, \tau)\, ds = \psi(s, \tau)$, moreover the solution of the corresponding boundary-value problems is singular during constant circulation or in quasi-stationary setting (with the fixed/recorded point of the descent of flow). Specifically, so is solved the already mentioned problem of the flow around the rotating circular gratings at constant angular velocity. If are examined small motions of lattices as a whole of their relatively steady flow (considering known), then problem still is simplified and is reduced, in linear setting relative to the supplementary unsteady speeds and pressures, to the calculation of quadratures in zone of flow or in the plane of parametric variable (in canonical area). In particular, all the kinematic formulas of the cascade theories of their fine/thin airfoil/profiles are directly used during assigned functions $v_y^+ = v_n^+(x, \tau), v_y^- = v_n^-(x, \tau)$, of the determined not only form airfoil/profiles, but also by their unsteady motion or deformation (the "hypothesis of dynamic curvature"). Actually, in this case in each the fixed/recorded moment of time is solved stationary problem with the specific value of the parameter τ.

In the basic case of the steady harmonic oscillations, the dependence of boundary conditions and all unsteady functions on time is isolated into the exponential factor

$$F(z, \tau) = F(z)\, e^{i\nu\tau}, \quad F(z) = F_1(z) + jF_2(z),$$

where ν - frequency, j - second apparent/imaginary "time/temporary" unity, which does not interact with the "three-dimensional/space" imaginary unit i, F (z) - composite (on j) amplitude of oscillations; with this problem it is reduced to definition of two functions (F_1 and F_2) of the composite (on i) variable z. Airfoil/profiles in lattice can oscillate synchronously, but with constant phase displacement α between airfoil/profiles.

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Then acts the condition of the generalized periodicity

$$F(z+imt,\tau)=F(z+imt)e^{j\nu\tau}=F_0(z)e^{j(m\alpha+\nu\tau)}\qquad (m=\pm 1,\ \pm 2,\ \ldots),\ (5.2)$$

where $F_0(z)$ - the composite amplitude, which relates to fundamental period (m=0), and in the integral representations analytic functions of the flow through the lattice nucleus $\operatorname{cth}\varkappa(\zeta-z)\ (\varkappa=\pi/t)$ is replaced by the more general

$$\Phi(\zeta-z,\varkappa,\alpha)=\frac{\operatorname{ch}\varkappa(1-\alpha/\pi)(\zeta-z)-ji\operatorname{sh}\varkappa(1-\alpha/\pi)(\zeta-z)}{\operatorname{sh}\varkappa(\zeta-z)},\qquad (5.3)$$

containing the same special feature/peculiarities and that satisfying condition (5.2) (G. S. Samoylovich, 1962). In particular, the composite amplitude of the supplementary speed, caused by the oscillations of airfoil/profiles and which disappears at infinity, are obtained the representations

$$\bar{v}_0(z) = \frac{1}{2\iota} \oint_C \bar{v}_0(\zeta)\, \Phi(\zeta - z)\, d\zeta, \tag{5.4}$$

$$\bar{v}_0(z) = \frac{1}{2\iota} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} c_{n+1} \Phi^{(n)}(z), \tag{5.5}$$

which, obviously, generalize formulas (3.1) and (3.8) for F (z) $=\bar{v}(z)$ and transfer/convert in them with $\alpha=0$. On the basis of the representation of the form (5.5) of function, which conformally reflects the assigned lattice to the equivalent lattice of circles, was solved the quasi-staticrary problem of the flow around the foil lattice of arbitrary airfoil/profiles (G. S. Samoylovich, 1962) and was calculated supplementary circulation for several typical turbine lattices with arbitrary phase displacement of the oscillations of adjacent airfoil/profiles (G. S. Samoylovich and B. E. Kapelovich, 1967).

Considerably more complex is the complete unsteady problem in which one must take into account free vortices, which descend into flow. As in the case of single airfoil/profile, this problem is solved only on the assumption that the motion of free vortices is known (it is usually considered that it coincides with particle motion on the critical flow line of steady flow).

Were first of all studied the steady-state oscillations of

fine/thin airfoil/profiles or plates with direct/straight after them. In accordance with the condition of conservation of vorticity in trace after airfoil/profile ($x \geqslant a$, $y=0$) are distributed the eddy/vortices with the intensity

$$\gamma(x, \tau) = \gamma(a, \tau') = -\frac{1}{U}\frac{d\Gamma}{d\tau}\Big|_{\tau=\tau'} = -j\frac{\nu}{U}Ae^{-j\nu\frac{x-a}{U}}e^{j\nu\tau}, \quad \tau' = \tau - \frac{x-a}{U}, \tag{5.6}$$

where U - the constant velocity of steady flow along x, x=a corresponds to the trailing edge of airfoil/profile, $\Gamma(\tau) = A \exp j\nu\tau$ - circulation around it, determined by Zhukovskiy - Chaplygin's condition about final speed with x=a.

The first tasks were examined in the beginning of the fiftieth years by G. Zengen (ZAMP, 1953, 4:4, 267-297), by F. Sisto (J Aeronaut. Sci., 1955, 22:5, 297-302), K. Nikelem (Ingr-Arch., 1955, 23:3, 179-188) et al. For solution were utilized the integral representations of speed on the airfoil/profile of form (5.4) (in the hydrodynamic interpretation - vortex/eddy method), by which the task one way or another is reduced to the solution of integral equation relative to vorticity distribution (tangential speed) on airfoil/profile or in trace. From the obtained vorticity distribution, unsteady forces are determined by direct calculation or by applying the common/general/total theorems. The foreign authors examined the tasks of the oscillation of plates in lattice upon different limited settings (large denseness, the absence of carrying

cut, cophasal of oscillaticr, etc.), moreover, either they generally avoided obtaining concrete/specific/actual results or were given only separate examples and particular dependences.

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The first systematic numerical results in the form of the dimensionless force coefficients and torque/moments, which act on plates in lattice during cophasal forward/progressive and rotary oscillations over a wide range of denseresses, angles of carrying out and frequencies, were published by S. M. Belotserkovskiy, A. S. Ginevskiy and Ya. Ye. Polonskiy in 1961. These results were obtained by the simplest method of discrete eddy/vortices.

As in stationary task, besides vortex/eddy method for a lattice from fine/thin airfoil/profiles there are more advanced analytical representations of the unknown functions of flow (L. I. Sedov, 1935-1939). The application/uses of these representations for the lattice of plates, during the steady cophasal oscillations appeared in 1958 in the works of M. D. Khaskind and T. K. Sirazetdinova and abroad - to I. L. Popesk (Comun. Acad. R.P. Romine, 1958, 8:10, 1003-1009).

In two works (pertaining to the year 1956) M. D. Khaskind,

examining the lattice of plates with carrying cut, and also the arbitrary system of cuts of one straight line, utilizes the method of solution, developed by it earlier in the task of the oscillation of fine/thin single airfoil/profile in the subsonic flow of gas (1947). The amplitude values of the composite potential of disturbance/perturbation are divide/marked off on two parts: w $(z)=w_0(z)+w_1(z)$; $w_0(z)$ is determined the noncirculating flow around lattice with an assigned normal speed of v_n, ~~of~~ and $w_1(z)$ corresponds to the solution of the uniform problem of the circulation flow around rigid lattice in the presence of free vortices. In order to find $w_1(z)$ that presents basic difficulty, is introduced analytic function

$$f(z)=\frac{dw_1}{dz}+j\frac{\nu}{U}w_1 \quad (5.7)$$

(analogous function was introduced in 1935 by M. V. Keldysh for the solution of two-dimensional problems of waves theory). In linear approach/approximation also during harmonic oscillations for overpressure disturbance/perturbation according to the equation of Koshi - Lagrange it is obtained

$$p=\rho U\left(\frac{\partial\varphi}{\partial x}+j\frac{\nu}{U}\varphi\right)e^{j\nu\tau}=\rho Ue^{j\nu\tau}\mathrm{Re}_{(i)}f(z),$$

therefore the introduced function f (z) with an accuracy to constant factor is equal to the amplitude of the composite (on i) potential of the accelerations of the flow in question. The real part of the potential of accelerations as pressure, is continuous (it is equal to

zero) in trace, apparent/imaginary part (in linear approach/approximation) it is known on airfoil/profile. Thus, functions $w_0(z)$ and f (z) are expressed by quadratures on the formulas of the cascade theories from fine/thin airfoil/profiles, $w_0(z)$ - is unambiguously, and f(z) and respectively $w_1(z)$ linearly depend on cyclic constants (circulation) which are located from the linear equations, which express the conditions of finiteness dw/dz on the trailing edges of airfoil/profiles. For the system of cuts of one by straight line (polyplane "tandem") and a lattice without carrying cut, the solution is constructed directly in flow plane z, for a lattice with carrying out, - in the plane of the parametric variable u=u(z), that corresponds to the representation of this lattice onto lattice without carrying cut.

The properties of the potential of accelerations indicated make with possible its application/use for the direct solution of unsteady problems. G. S. Samoylovich in 1961, it found by this apparent-mass method and the distribution of pressures in the lattice of plates without carrying out with the arbitrary forms of their oscillations through one, moreover were utilized series from functions of the type of the composite rate of the flow around the lattices of plates, and also (during cophasal oscillations) the integral formulas of slender-wing theory.

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T. K. Sirazetdinov and I. L. Popesk in 1958 generalized L. I. Sedov's method in the problem of the unsteady motion of single fine/thin airfoil/profile in motionless liquid to the case of cophasal lattice vibrations from plates without carrying out with rectilinear vortex/eddy after them, on the basis of the integral representation of composite perturbation rate

$$\bar{v}(z, \tau) = \frac{1}{2\pi i g(z)} \left\{ \int_{-a}^{a} 2v_n(\xi, \tau)\, g(\xi)\, [\mathrm{cth}(\xi - z) + 1]\, d\xi + \int_{-\infty}^{-a} \gamma(\xi, \tau)\, g(\xi)\, [\mathrm{cth}(\xi - z) + 1]\, d\xi \right\}, \quad (5.8)$$

where $g(z) = \sqrt{\mathrm{sh}(z-a)/\mathrm{sh}(z+a)}$, the period of lattice $t=\pi$. This representation automatically, because of the properties of $g(z)$, satisfies the Zhukovskiy - Charlugin condition on trailing edges $z=-a+im\pi$ and taking into account communication/connection of type (5.6) between γ and $\bar{v}(x)$ reduces to integral equation relative to function $\gamma(x)$ which for the steady harmonic oscillations simply is solved and it makes it possible to express $\gamma(x)$ and with respect to $\bar{v}(z)$ in quadratures.

Representation of (5.8) by use in the nucleus of function Φ

(ζ-z; α) (5.3) is generalized to the case of harmonic oscillations with the constant shift α of the phases of the oscillations of adjacent airfoil/profiles (G. S. Samcylcvich, 1962), and during the use cf expansion of (5.5) and of Chaplugin - Sedov's formulas, it makes it possible to obtain the common/general/total expressions of the composite amplitudes of unsteady forces and torque/moments in the form of final formulas (quadratures) whose each member makes specific hydrodynamic sense (V. P. Vakhomchik, 1965, 1966). Such expressions have some computational advantages before the simplest vortex/eddy method and, furthermore, make it possible analytically to obtain for the limiting values of the geometric and kinematic parameters the asymptotic results which, as a rule, they slip off from numerical calculations.

For a lattice with the angle of forward projection $\beta \neq 0$ of function g(z) and $\Phi(\zeta - z)$ it is possible to generalize by the introduction of composite parameter $\varkappa_1 = \varkappa \exp(-i\beta)$ instead of $\varkappa = \nu/t$; however, in a representation of type (5.8) in this case is retained unknown function $\gamma_1 = \nu_x^+ - \nu_x^-$ (equal to zero with β=0), and it gives only the isolation/liberation of special feature/peculiarities $\bar{v}(z)$, which facilitates investigation and the solution of the corresponding integral equation.

Unlike stationary problem, obtaining systematic numerical

results in the unsteady problem in question is very laborious and in practice it is inaccessible without use of ETsVM [ЭЦВМ - digital computer]. On the other hand, computer technology makes it possible to realize some specific methods whose use for a manual count would be unsuitable, and, in particular, the method of aerodynamic interference. In accordance with this method the velocity potential of the flow around the system of bodies in question occurs in the form of the sum of the potentials of the flow around each of these bodies separately, that moves (and, generally speaking, that is deformed) along the assigned law, and the supplementary potentials, determined by some supplementary previously unknown laws of strains whose introduction considers the interference of bodies.

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For determining these supplementary strains, is utilized the condition of the impenetrability of bodies, which gives system of equations which are solved by the method of trigonometric polynomials or by the method of collocation. Circulations around bodies are determined by Zhukovskiy - Chaplygin's condition; the form of vortex wakes is considered known or it enters in solution implicitly, if is utilized the potential of accelerations (5.7) (D. N. Gorelov, 1963, 1964).

By the method of interference were studied the oscillations of biplane and airfoil/profile in flow with different (rigid and free) boundaries (D. N. Gorelov, 1964, 1965), and also the forward/progressive and rotary oscillations of plates in lattice - for the first time over a wide range of a change in all geometric and kinematic parameters. (In the latter case instead of the lattice actually was taken system from a sufficiently large finite number of airfoil/profiles). In connection with this method it proved to be natural to find the influence coefficients, determining unsteady forces on one airfoil/profile during small motion (oscillation) along the assigned law only of one another body (V. B. Kurzin, 1964; D. N. Gorelov, 1964, 1965). In the case of lattice influence coefficients can be defined as Fourier coefficients in the resolution of dimensionless aerodynamic forces in terms of the phase angle of the oscillations of adjacent airfoil/profiles (V. B. Kurzin, 1964; G. S. Samoylovich and B. E. Kapelovich 1967), also, in any event - directly in terms of the method of interference (D. N. Gorelov, 1964, 1965). After are found the influence coefficients, via superposition are simply determined loads on the airfoil/profiles, which vary with different amplitudes and phases, but with identical frequencies and the forms of the oscillations (limitation of identical forms is unessential).

The problem of the oscillations of arbitrary lattice, as has

already been indicated, is solved most simple (by quadrature) in quasi-stationary setting, i.e., without taking into account vortex wakes in flow after airfoil/profiles. The calculations, carried out for the lattice of plates, showed that this examination is virtually admissible (vortex wakes barely affect) in the lattices of large denseness, and also at small frequencies, if phase displacement $\alpha \neq 0$ (S. M. Belotserkovskiy et al., 1961; G. S. Samoylovich, 1962; D. N. Gorelov, 1964). Analogously it is possible to solve this problem, if is accepted another model of vortex wake after airfoil/profiles in the form of the infinite cut/section of known form (G. Yu. Stepanov, 1962), of the stationary or deforming in accordance with oscillation airfoil/profile.

The solution of complete unsteady problem for an arbitrary lattice in principle is possible by the same methods which were applied for the lattice of plates, namely by vortex/eddy, the potential of accelerations and interference, moreover calculations become complicated by the need for integrating by the duct/contour of airfoil/profile C, but not on the line segment. During the study of this problem, was establish/installed the presence of the effect of the final displacement of airfoil/profiles (besides the rate of this displacement). The effect of final bias was for the first time estimated based on the example of the lattice of the plates, varying with phase displacement during steady flow with sizable angle of

attack (V. V. Musatov, 1963). In quasi-stationary setting or during the use of a model with cut/sections beyond airfoil/profiles this effect is located as effect of small strain of airfoil/profile in stationary nonuniform flow; in complete unsteady setting occurs the corresponding complication of the integral equations of problem (V. E. Saren, 1966). V. B. Kurzin in 1967 it outlined new approach to the solution of this problem with the aid of the method of "cementing", according to which entire/all zone of flow through the lattice is divided into three subregion: the incident flow, vane channel and flow after lattice; in each of the subregions, is solved the corresponding problem relative to velocity potential taking into account the conditions of its continuity on the boundaries between subregions.

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This approach deserves attention, first of all, for the dense lattices when it suffices to examine only vane channel with approximate boundary entrance conditions and output and it is possible not to consider the presence of special feature/peculiarities on edges.

During the oscillations of bodies in the flow of gas, appear the essential effects, connected with the final velocity of propagation

of disturbance/perturbations. According to Prandtl this problem is examined, in linear acoustic approach/approximation, by the introduction of the potential of accelerations, continuous in zone of flow and satisfying wave equation; the determination of unsteady pressures on the oscillating single airfoil/profile at subsonic speeds is reduced in this approach/approximation to the solution of the integral equation of Possio with the nucleus, which contains the functions of Hankel (see A. I. Nekrasov, 1947). As V. B. Kurzin in 1962 it generalized the equation of Possio to the case of the lattice of plates and in 1964 was obtained its approximate numerical solution by the method of collocation with the propagation of solution by the introduction of influence coefficients on the case of the arbitrary oscillations of plates (1966). Based on the example of lattice without carrying out, were shown the essential dependence of unsteady forces on number $M = U/v_{\infty}$ of stationary flow and the presence of phenomena of the type of resonances at frequencies, multiple to the natural frequencies of oscillation of gas in the vane channel of lattice in the longitudinal and transverse directions, respectively equal to approximately $\nu_1 = \pi v_{\infty}(1 - M^2)/l$ and $\nu_2 = \pi v_{\infty}/t$. D. N. Gorelov in 1963 used the method of aerodynamic interference to the more common/general/total problem of subsonic spatial flow of the gas through lattice of the plates, varying between two parallel planes. During the construction of solution, was utilized velocity potential of the flow around single plate in plane flow, expressed M. D.

Khaskindom (1947), actually with the aid of the method of the potential of accelerations, in the elliptical coordinates through Mathieu - Hankek functions. The infinite system of equations, which expresses the conditions of the impenetrability of plates, was solved by the method of collocation. Thus carried out calculations for flat/plane and spatial flow (D. N. Gorelov and L. V. Dominas, 1966, 1967) and was shown the possibility of the auto-oscillations of blade with one degree of freedom during the near-resonant conditions/modes, corresponding, besides indicated above, to the natural frequencies of oscillation of gas along the forming three-dimensional/space blades with a length of h (between the limiting planes) $\nu_3 = \pi v_{3B}/h$, and also still to some frequencies ν_4, depending on the angle of carrying out β and of the shift α of the phases of the oscillations of adjacent blades. The last/latter frequencies, which depend on interaction of blades, coincide with the eigenvalues of matrix of linear equations and they are physically treated by the authors as imposition of the cophasal disturbance/perturbations from adjacent blades. These frequencies for plane flow for the first time and by most direct/straight method were determined in 1967 G. S. Samoylovichem, who examined the propagation of waves along the axis of the lattice of the dipoles, corresponding to the action of concentrated forces, and flow behavior far from lattice. In subsonic flow (with M<1)

$$\nu_4 = (\pm M \sin\beta + \sqrt{1 - M^2\cos^2\beta})\,\frac{\alpha v_{3B}}{t},$$

in supersonic (when 1<M<1/cos β),

$$\nu_4 = (M \sin\beta \pm \sqrt{1 - M^2\cos^2\beta})\,\frac{\alpha v_{3B}}{t}.$$

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(Questicn concerning the amcunts of forces under conditions of resonances is permitted from acoustic positions or by special asymptotic experiment).

As a result of the solution of spatial problem, were shown the admissibility of the hypothesis of flat/plane sections for the incompressible fluid and the presence of essential deflections from plane flow for gas during any real elongations of blades.

Lattice vibrations in the supersonic flow represent a simpler problem, solved by the general methods, developed for the arbitrary system of finite-span wings (E. A. Krasil'shskikov, 1952). They concrete/specific/actually studied, by the method of aerodynamic interference, lattice vibration of plates without carrying out with phase displacement $\alpha=0$ and $\alpha=\pi$ (which schematizes the oscillations of single plate respectively in free jet and between rigid walls) with super and at transonic speeds (D. N. Gorelov, 1966).

Viscosity effect during lattice vibrations theoretically barely was studied and was drawn cnly for the schematic explanation of

experimental results (G. S. Samoylovich, 1963-1967; V. N. Yershov, 1966). This effect leads to the appearance of unsteady (" rotating") flow separation, decrease, phase lag and hysterisis of unsteady forces, especially at high frequencies, and to fading of vortex wakes after lattice.

The common/general/total problem of the aeroelasticity of the blades of turbomachines is reduced to the study of the equations of their motions which for a linear dynamic system with a finite number of degrees of freedom can be written in the form of the matrix equation

$$A\ddot{X}+B\dot{X}+CX=F, \qquad (5,9)$$

in which X - matrix/die of generalized coordinates (strains), of A - masses, B - coefficients of friction (attenuations), of C - elasticity, F - exciting forces. The mechanical feature of the problem, which concerns free oscillations without taking into account of interaction of blades with medium, is relatively simple and studied in sufficient detail (for example, for the turbine blades, see A. V. Levin's monograph, 1953). In the examined above hydroaerodynamic problems the motion of blades $X = X_0 \exp j\nu\tau$ was considered known (X_0 - the matrix/die, assigning form and phase displacements of oscillations) were determined the forces of pressure flow, depending on acceleration, speed and the bias of blades. According to relation to an entire problem in these problems, are

more precisely formulated the matrix/dies of the coefficients of equations of (5.9) because of apparent additional masses, aerodynamic damping, position forces and additional constraints (aerodynamic interference of oscillations). As a result it becomes possible to refine natural frequencies and the forms of the oscillations of blades, to determine the limits of stability and auto-oscillations (flutter). The appearance of the latter is determined by the aerodynamic interference of the various forms of the oscillations of one blade (classical flutter), of the blades between themselves (latticed flutter), resonances in gas, sonic "closing" and finally by flow breakaway (stalled flutter). In this case, are especially important the forces, which are found in phase (or in antiphase) with flutter speed on each main form, including forces of aerodynamic damping, connected with descent into the flow of eddy/vortices, emission/radiation in gas and frictional forces in viscous fluid (G. S. Samoylovich, 1962; L. E. Ol'shteyn and F. A. Shipov, 1962-1967, etc.).

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For the definition of strains and the dynamic stresses in blades, it is necessary to know exciting forces of F as explicit periodic functions of time. Low-frequency excitations depend on the properties of grid/network, control system, characteristics of

turbomachines and mode of their operation. These excitations were studied in one-dimensioral (and quasi-staticnary with respect to turbcmachine) setting by the methods of the thecry of nonlinear vibrations (V. V. Kazakevich, 1959).

Radio-frequencys drive depend cn relative motion of the lattices and other bodies, which are located in flow, for example fastening cell/elements. The corresponding forces are connected with the heterogeneity of flow and relatively simply they are located in quasi-stationary setting for the potential flow around lattices and other systems of bodies (L. A. Dorfman, 1947; G. Yu. Stepanov, 1962; V. P. Vakhomchik, 1962). However, heterogeneities in the potential flow rapidly attenuate (for lattices as index of the distance between them) and, as a rule, they cannot be the basic reason for excitation. Main role play the vortex wakes in the incident flow, form and intensity of which are determined by the viscosity of liquid and by flow turbulence. Within the limits of lattice these traces to admissibly consider as vortex/eddy heterogeneities in the flow of nonviscous liquid. With a small heterogeneity the definition of its effect is reduced by known form as in the problem of wing in vortex/eddy "gust", taking into consideration to the supplementary rate of deformation of airfoil/profile in uniform potential flow (G. S. Samcylovich, 1961, 1962). With large heterogeneity and taking into account interaction of lattices this problem is very complex; are

known some experimental investigaticns in quasi-stationary setting and the one-dimensional estimations of forces according to maximum.

In the majority of investigations, were examined uniform lattices to the identical cenditicns of flcw and excitation of identical blades. The study of the aeroelasticity of mechanically heterogenecus circular lattices (L. E. Ol'shteyn and R. A. Shipov, 1962-1967; V. B. Kurzin, 1967) conducted shcwed that such lattices are dynamically more stable in flow than uniform, having, however, higher vibration stresses.

As a whole in the range of the theory of the unsteady flow of lattices in our country, were cbtained imFcrtant results, as a rule, the anticipate/leading analogcus results in foreign investigations. In a number of these results develcpment of new theoretical methods, cbtaining systematic calculaticn data, the establishment of new effects and the indication cf the ways cf further experiments. In recent years especially intense works are ccnducted in the Moscow Power Engineering Institute and in the Institute of Hydrodynamics of the Siberian Department of the Academy cf Sciences of the USSR; based cn materials of these wcrks, are prepared two special monographs (G. S. Samoylovich; D. N. Gcrelov, V. B. Kurzin and V. E. Saren).

§6. Three-dimensional/space cascade flow.

Real flow in the space lattices of turbomachines essentially three-dimensional and unsteady. The investigation of this flow taking into account the effects of imperfect gas is the basic contemporary problem of the cascade theories, to the need for solution of which they feed both the overall development of this theory and a series of the new problems, which appear during design and use of turbomachines. Only complexity of the direct solution of the corresponding problems forces to turn to the simplified models of the flow of the idealized liquids and with a smaller number of independent variables.

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The application/use of the simplified models of three-dimensional/space flows in the theory of turbomachines is justified, however, even because for practical target/purposes is not usually necessary the perfect information about all field of flow and are sufficient only common/general/total characteristic parameters whose determination with satisfactory accuracy is conducted on stationary two-dimensional or even one-dimensional models.

Such models successfully applied N. E. Joukowski, who was using

the hypothesis of cylindrical sections for transitioa to foil lattice (1890, 1912-1914) and by the hypothesis of connected and free vortices (1906) in his investigations of screw propeller and windmill (1912-1915). In these investigations Jcukowski for the first time, considerably earlier than the foreign authors, introduced the vortex/eddy models of screw/propeller with final and with an infinite number of blade/vanes ("vortex/eddy base"), proposed the blade/vane of constant circulation (screw/propeller "NEZh"), examined twisted axisymmetric flows after screw/propeller, which more were lately called the "laws of torsicn" with the constant moment of momentum (rv_φ =const), by a constant flow angle (v_φ/v_z = const) and of the type of solid body (v_φ/r = const).

Vortex conception of screw/propeller was the great theoretical achievement, which made it possible to substantially refine its theoretical model and to base the widespread hydraulic (one-dimensional) calculation of screw/propeller [1].

FOOTNOTE [1]. In the known previously theoretical model of screw/propeller, each cross section of propeller blade was considered as isolated/insulated airfoil/profile in flat/plane relative flow (S. K. Dzhevetskiy, 1892). ENDFOOTNOTE.

It should be noted that the initial vortex/eddy model of

screw/propeller with a finite number of blade/vanes included as idle time a special case, the vortex/eddy model of finite-span wing, proposed almost simultaneously to S. A. Chaplygin, by F. Lanchester and L. Prandtl in 1911. Subsequently the development of the linear airfoil theory of final spread/scope, were obtained important results for the incompressible fluid and gas at subsonic speeds (N. E. Kochin, 1941; V. V. Golubev, 1939-1947; V. V. Struminskiy, 1946-1957; S. M. Belotserkovskiy, 1955, 1965) and at supersonic speeds (E. A. Krasilsyuchyuikov, 1947-1952; S. V. Falkovich, 1947; M. I. Gurevich, 1947; L. A. Galina, 1947).

Vortex conception of screw/propeller was successfully develop/processed by N. Ye. Joukowski's school and were obtained numerous practical application/uses to air and screw propellers, windmill motors, fans and blowers (V. P. Vetchinkin and N. N. Polyakhov, 1940; L. A. Simonov and S. A. Khristianovich, 1944; F. I. Frankl, 1946; B. N. Yur'yev, 1956, 1961; A. M. Basin and I. Ya. Miniovich,1963). Direct relation to the theory of turbomachines has the problem of screw/propeller in the encircling ring, proposed in 1864 by K. F. Briks (I. V. Ostoslavskiy and V. N. Matvei, 1935; M. B. Maseyev and M. N. Veselovskiy, 1946). For the theory of turbomachines, is interesting also L. A. Simonov's work (1941), in which the vortex/eddy method of the construction of blade/vane in plane flow was common for the case of the radial eddy/vortices

between two coaxial circular cylinders and was shown a small error in the hypothesis of cylindrical sections.

Thus, already in the works of N. Ye. Joukowski and his school the three-dimensional problem of the theory of turbomachines was considered as two two-dimensional - the problem of neutral axisymmetric vortex flow and the problem of the flow through lattices in the axisymmetric (in particular, cylindrical) section of blade/vanes.

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In the range of hydroturbines, G. Lorenz (Neue Theorie und Berechnung der Kreiselräder, 1906) hypotheticalally introduced the vortex/eddy axisymmetric model of the flow through the space lattice from an infinite number of infinitely fine/thin blade/vanes with the distributed (mass) force of their effect on flow. On the basis of V. Bauyersfel'd's this model in 1907, it solved the inverse axisymmetric problem of the construction of the blade/vane of radial-axial turbine for a special case of irrotational meridional flow $\mathrm{rot}_{\varphi} v = 0$. I. N. Voznesensky during the years 1920-1935 developed method of the construction of vortex flow which was used for the design of the first Soviet hydroturbines (I. N. Voznesensky, 1952). Hydrodynamic methods in hydro-machine-building received our country's first

systematic presentation and development in the courses of A. A. Satkevich (1929), of I. N. Voznesersky (1934), of G. F. Proskury (1934).

In the range of steam turbines and aircraft gas-turbine engines the conventional one-dimensional calculation in postwar years was supplemented by the already mentioned "laws of the torsion" of cylindrical $(v_r = 0)$ axisymmetric flow of gas with the constant moment of momentum $rv_\varphi = \text{const}$ (V. V. Uvarov, 1945), constant flow angle $v_\varphi/v_z = \text{const}$, by constant specific flow rate/consumption $\rho v_z = \text{const}$ and by others (B. S. Stechkin et al., 1956; L. A. Simonov, 1957; G. Yu. Stepanov, 1958; E. A. Gukasova et al., 1958; K. A. Ushakov et al., 1960; G. N. Abramovich, 1953; M. E. Deutsch, 1953; L. I. Cyril, 1964), moreover approximately were considered the noncylindrical form of flow area and alternating/variable on a radius the total pressure and the temperature. The calculation of the axisymmetric twisted flow in the clearances between the motionless and revolving gates made it possible to obtain the necessary data for the shaping of blades on height/altitude and for determining the common/general/total indices of turbomachine. Since the latter is conducted within the framework of one-dimensional model, then proved to be necessary the refinement of the concept of the ideal one-dimensional process and average parameters in the characteristic sections of flow (G. G. Chernyy, 1953, 1956; L. I. Sedov and G. G. Chernyy, 1954; G. N. Abramovich,

1953).

The necessary supplements indicated and the refinements of one-dimensional calculations were conducted and were realized in the construction/designs of engines virtually simultaneously and it is independent in Soviet and foreign works.

In the beginning of the fiftieth years in the USA, were published the first calculations of the two-dimensional models of spatial flow of inviscid compressible liquid through turbomachines (Ch. Kh. Vu, Trans. Amer. Soc. Mech. Engnrs, 1952, 74:8, 1363-1380; J. D. Stanits, there, 74:4, 473-497). In the analogous investigations, which were being carried out in our country, special attention was allotted to the fundamental side of a question, to the refinement of the hydrodynamic formulation of the problems, to the substantiation of the simplified models and calculation methods. For this, was required the application/use of common/general/total theory of vortices of the motions of gas (A. A. Friedman, 1921, 1932; N. E. Kochin et al., 1953) and, in particular, in systems with rotating communication/connections (V. M. Astaf'yev, 1949), the study of the geometric properties of flow (S. S. Byushgens, 1948, 1951) and the use of a process/operation of the averaging of equations (G. Yu. Stepanov, 1952; Ya. A. Sirotkin, 1963, 1967).

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For obtaining the equations of the stationary axisymmetric flow through the turbomachine of the inviscid ideal gas to common/general/total equations of motion, is applied linear averaging in circumference and on time, what gives (all parameters and the operators - averaged)

$$\left.\begin{aligned} w\times(\nabla\times v)=\nabla H-T\nabla S-F+R_1, \\ \nabla\cdot(\chi\rho w)=R_2,\quad p=R\rho T+R_3, \end{aligned}\right\} \qquad (6.1)$$

where besides usual designations are accepted following: w - relative speed, $v=w+u=w+\omega\times r$ - absolute velocity, ω=const - angular rate of rotation of lattice, $H=J+1/2w^2-1/2u^2$, $J=c_pT$ - enthalpy, $dH=0$, S - entropy, $dS=c_v dp/p-c_p d\rho/\rho=0$. As a result of averaging in equations (6.1), unlike the usual equations of axisymmetric flow, enter coefficient χ of the constraint of flow in circular direction by the blades of final thickness, the mass distributed force F of the effect of blades on flow, the quadratic pulsating terms R_1, R_2 and R_3 - function of a change of the flow parameters in circular direction and in time. Pulsating terms disappear in axisymmetric flow and in model with an infinite number of blades. System of equations (6.1) is closed by the relationship/ratios

$$v\times F=0,\quad v\cdot w=0, \qquad (6.2)$$

expressing respectively collinearity F and orthogonality w of standard v of average stream surface.

For the precision determination of the members of averaging χ, F and of the form of average stream surface, it is necessary to know the complete three-dimensional variable field of flow; however virtually, with an accuracy down to the terms of the order of the square of pulsations (in circular direction and on time), functions χ and ν are determined by geometric parameters of vane channels, but the pulsating terms F are omitted. (This usually adopted simplification corresponds to the model of the flow through the turbomachine with an infinite number of blades whose thickness is considered by the coefficient of constraint $0<\chi<1$.) For the estimation of error in the simplified axisymmetric model and refinement of calculation, it is possible to utilize theory of the two-dimensional flow around lattices on axisymmetric stream surface and theory of secondary flows.

It is also the possible to consider small effects of real flow (viscosity, thermal conductivity, phase transitions) by the phenomenological introduction of the axially symmetric fields of small frictional forces, sources of heat and mass with the addition of the equations of energy ($dH\neq0$), of entropy ($dS\neq0$) and of diffusion.

For solution initial equations are record/written in meridional plane in natural (s, n) or that fix/recorded, for example, by cylindrical (r, z) the coordinate systems. In the first case the coordinate system is previously unknown and equations are supplemented by the relationship/ratio of the orthogonality

$$\frac{dK_s}{dn}-\frac{dK_n}{ds}=K_s^2+K_n^2, \quad (6.3)$$

where K_s and K_n - curvatures respectively of the flow lines s and of the normal to them lines n, the derivatives d/ds and d/dn are taken along the appropriate lines, $K_n=-d\ln(\chi r\rho v_m)/ds$, v_m - a projection of speed on meridional plane (G. Yu. Stepanov, 1962).

In the second case, in the fixed/recorded coordinate system, the system of equations is reduced either to twos relative to the projections of the speed on the coordinate axis or to one equation relative to the function of current in direct problem (G. I. Maykapar, 1958; P. A. Romanenko, 1959; Ya. A. Sirotkin, 1963-1967) or relative to the function $\phi(r, z)$, which determines average stream surface in inverse problem (I. N. Voznesensky, 1952; Ya. A. Sirotkin, 1966).

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In the particular case of the incompressible fluid and in cylindrical coordinate system this equation takes the form

$$\left[1+r^2\left(\frac{\partial\varphi}{\partial z}\right)^2\right]\frac{\partial^2\psi}{\partial r^2}-2r^2\frac{\partial\varphi}{\partial r}\frac{\partial\varphi}{\partial z}\frac{\partial^2\psi}{\partial r\,\partial z}+\left[1+r^2\left(\frac{\partial\varphi}{\partial r}\right)^2\right]\frac{\partial^2\psi}{\partial z^2}-$$
$$-r^2\left[\frac{\partial\psi}{\partial z}\frac{\partial\varphi}{\partial z}\frac{\partial^2\varphi}{\partial r^2}-\left(\frac{\partial\psi}{\partial z}\frac{\partial\varphi}{\partial r}+\frac{\partial\psi}{\partial r}\frac{\partial\varphi}{\partial z}\right)\frac{\partial^2\varphi}{\partial r\,\partial z}+\frac{\partial\psi}{\partial r}\frac{\partial\varphi}{\partial r}\frac{\partial^2\varphi}{\partial z^2}\right]=$$
$$r^2\chi^2\frac{dH}{d\psi}+f\left(r,\ z,\ \frac{\partial\psi}{\partial r},\ \frac{\partial\psi}{\partial z},\ \frac{\partial\varphi}{\partial r},\ \frac{\partial\varphi}{\partial z}\right)\quad\left(H=\frac{p}{\rho}+\frac{w^2}{2}-\frac{u^2}{2}\right).\qquad(6.4)$$

In direct problem equation (6.4) has relative to function ψ (r, z) elliptical type, in inverse problem - hyperbolic relative to function ϕ (r, z). In free from lattices annular channel is obtained the elliptic equation of the generalized helical motion (I. S. Gromeka, 1882; O. F. Vasiliev, 1958)

$$\frac{\partial^2\psi}{\partial z^2}+\frac{\partial^2\psi}{\partial r^2}-\frac{1}{2}\frac{\partial\psi}{\partial r}=r^2\frac{\partial H}{\partial\psi}-\frac{1}{2}\frac{d}{d\psi}(rv_\varphi)^2\quad\left(H=\frac{p}{\rho}+\frac{v^2}{2}\right).\qquad(6.5)$$

The analogous equation of direct problem for compressible liquid elliptically relativeto ψ at subsonic speeds in meridional plane $v_m < v_{зв}$ (supersonic flows $v_m > v_{зв}$ in internal problems are unstable and, as a rule, they are not realized). In connection with the type of equation, the boundary conditions are placed on band edges as for a channel. On upper and lower boundaries are assigned the Dirichlet conditions, at entrance and output, - directional derivative (slope/inclination of flow lines). Domain of definition ψ is divided on the subregion of three forms: the containing revolving gates, which contain rigid lattices and free from them. On the boundaries of subregions, generally speaking, occurs the final discontinuity/interruption of the coefficients of the higher derivatives; therefore, solution can exist in the class of the generalized functions with the integrated square. This, direct

axisymmetric problem is reduced to the mixed boundary/edge in simply connected region for quasi-linear elliptic equation with a finite number of lines of discontinuity of coefficients. For the incompressible fluid equation (6.4) in question is uniformly elliptical, and questions of existence and uniqueness of its solution are well studied (O. A. Ladyzhenskaya and N. N. Ural'tseva, 1964). For compressible liquid this equation, as well as all the equations of gas dynamics, is elliptical only during the limited together with first-order derivatives solution, and a question of its single-valued solvability even in simpler problems remains opened. By analogy with the theory of the plane subsonic flow of gas, it is possible to expect the existence of unique solution during the specific limitations of the right side of equation or gradients of functions H and S. These questions have not only fundamental, but also direct practical value, being connected with the stability of the twisted flows and with the convergence of iterative diagrams during the determination of numerical solutions.

Inverse axisymmetric problem in the traditional setting of Bauersfeld - Voznesensky consists in the determination of the form of the average surface of blade/vane $\varphi=\varphi(r, z)$ with the assigned function of current $\psi(r, z)$ or in assigned field of meridional velocities $v_m(r, z)$. Since for this problem equation (6.4) (and analogous for the flow of compressible liquid) has hyperbolic type,

then for it are placed the problems of Goursat and three mixed, if only the boundary of the subregion, which contains lattice, does not coincide with the line of parabolic degeneration of the type of equation.

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This line appears with the circular/neighboring projection of speed $w_\varphi = 0$ ($v_\varphi = 0$ for a rigid lattice), moreover, during transition through the line of degeneration within the limits of lattices equation its type is not changed. Thus, for an inverse problem is constructed the usual solution, analogous to the classical solution of supersonic problems of gas dynamics, with that difference, that in the vicinity of the line of parabolic degeneration the method of characteristics is directly inapplicable and on the boundaries of the subregions, which do not contain lattices, the solution must be mated with the solution of elliptic equation (6.5) (Ya. A. Sirotkin, 1964, 1966).

From foregoing, it is evident that direct and reverse/inverse problems of averaged axisymmetric flow obtained in our country common/general/total and in known sense final setting.

In the fiftieth years the solution of direct problem begins to be introduced in the practice of calculation and design of

turbomachines and obtains numerous examples of application/use. The solution of the problem of the relatively comprising speeds is conducted usually according to the method of straight lines and is reduced to the sequence of boundary-value problems for the system of ordinary differential equations in natural grid with the use of curvatures (G. Yu. Stepanov, 1953, 1962) or in that semifixed and in by that fix/recorded grids (L. A. Simonov, 1950, 1957; Ya. A. Sirotkin, 1959-1963; N. I. Durakov and C. I. Novikova, 1963; M. I. Joukowski, 1967). The solution of problem relative to the function of current is obtained by net point method (G. I. Maykapar, 1958; Ya. A. Sirotkin, 1964) or by variational method of Galerkin (P. A. Romanenko, 1959). In all cases, due to the nonlinearity of problem, are applied successive approximations, moreover, their convergence is checked or is reached (via the selection of the spaces of grid or weight coefficients) with the aid of numerical experiment. Calculations in the common/general/total formulation of the problem are proved to be very laborious and are oriented in essence for the application/use of contemporary ETsVM.

The solutions of inverse axisymmetric problems, after I. N. Voznesensky's mentioned above works, only begin to appear in the approximate and still inadequate setting (I. E. Etinberg, 1965; I. A. Oganesyan, 1967).

In technical practice won acceptance different, partially already mentioned, approximate methods of the solution of axisymmetric problems, instituted on different simplifying assumptions and the form of flow lines, such, for example, as theory of cylindrical and conical step/stages within the limits of the clearances between lattices (taking into account and without taking into account of the curvature of flow lines). All these methods are contained as special cases in the fundamental equations of axisymmetric problem and at a cost of the losses of the strictness of setting make it possible of obtaining the foreseeable solutions, which do not require the application/use of ETsVM (G. N. Abramovich, 1953; M. Ye. Deutsch and G. S. Samoylovich, 1959, etc.).

On the other hand, deserve reference examples of the construction of the three-dimensional/space (three-dimensional) flows of the incompressible fluid in the turbomachine, limited by spherical surfaces (A. M. Gokhman, 1954), and in vane channel (A. F. Makarov, 1967). Such very laborious calculations have, however, systematic value for estimating the accuracy of the simplified models.

The equations of the second two-dimensional task - the steady flow of lattices on surface of revolution in the layer of alternating/variable thickness - are obtained as a result of the averaging of common/general/total equations for time and across the

layer whose boundaries are considered coinciding with stream surfaces of axisymmetric flow r=r(z).

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Unlike the equations the first - axisymmetric - task are necessary supplementary assumptions about the smallness of thickness h=h(x) layer (on comparison of c r), and also about the smallness of the values of the transverse with respect to layer projection of speed v_n and of the gradients of all functions across layer (G. Yu. Stepanov, 1962; O. F. Vasiliev and N. S. Khapilova, 1965). For convenience in the image and calculations expedient to pass from the axisymmetric surface r=r(z) (dr/dz=tg γ) to the plane of its conformal mapping $x = \int_0^z dz/(r\cos\gamma),\ y = \varphi,$ in which any lattice is converted into straight line, arrange/located along y axis. As a result of averaging and neglect of low pulsating terms and the transversing speeds of equation of motion they are reduced to the equations of continuity and eddy/vortices in plane layer of alternating/variable thickness relative to the components relative speed $w_x = w_m,\ w_y = w_\varphi$:

$$\left.\begin{aligned}\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} &= 0,\\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} &= \Omega,\end{aligned}\right\} \tag{6.6}$$

or to equation relative to the function of current ψ:

$$\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial y}\right) = -\Omega, \tag{6.7}$$

where p=h(x) ρ(w, x) - the given thickness of the layer, $\rho=\rho(w, x)$ - gas density, which is known function on Bernoulli's surface (H=const, S=const), $u = rw_x = p^{-1}\partial\psi/\partial y,\quad v = rw_y = -p^{-1}\partial\psi/\partial x, \Omega = -2\omega r^2 \sin\gamma, \omega$ - the angular rate of rotation of lattice.

Equation (6.7) is elliptical at subsonic speeds $(w < v_{зв})$, and the formulation of the problems for it is analogous to the formulation of the problems of the subsonic flow around the foil lattices, which correspond to flow in the cylindrical layer of constant thickness (to hypothesis of cylindrical sections); r=const (γ=0), h=const. For the numerical solution of equations (6.6) and (6.7) in principle are suitable all methods, which were being mentioned in connection with the axisymmetric task, with respect to which these equations are idle time a special case, with that difference, that on the band edges of period out of lattice are placed periodicity conditions of flow and which in the vicinities of critical points should eliminate special feature/peculiarities (coinciding with the limited derivatives dh/dx with the appropriate special feature/peculiarities of flat/plane incompressible flow).

In an important special case ρ=const and Ω=0 (the second is unessential) equations (6.6) and (6.7) become linear and

transfer/convert into the well known equations of mathematical physics, which describe the motion of the electric current through the carrying out surfaces of arbitrary form (N. A. Umov, 1875), the flow of the incompressible fluid in the layer of alternating/variable thickness and laminar filtration in heterogeneous layers (O. V. Golubyov, 1950, 1953; P. Ya. Polubarinova-Kochina, 1953), the flow of gas in the hodograph plane of the speed (L. S. Leybenzon, 1935), the flow of viscous fluid in bearing, the stressed state of anisotropic shafts and heterogeneous plates. The mathematical theory of these equations is significantly developed in the works by I. N. Vekua, L. Bers and A. Vaynshteyn, M. A. Lavrentyev and B.V. Shabat, S. Bergmann, G. N. Polozhiy.

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The effective solutions of boundary-value problems for equations (6.6) and (6.7) occur through analytic (harmonics) functions and fundamental particular solutions, in hydrodynamic interpretation - by the imposition of the potentials of basic elementary flows of the type of the isolated/insulated sources, eddy/vortices, dipoles and multipoles. These particular solutions have known expressions (generalized axisymmetric potentials): for exponential layers $h = x^n$ in the general case through the Bessel functions on the order of 1/2 (n-1), with whole even n - through elementary functions and with

whole odd n - through elliptical integrals; for the exponential layers h=exp x and their linear combinations, including of trigonometric and hyperbolic functions, solution they are expressed as the functions of MacDonald. (In gas dynamics were studied in detail cases $h=\sqrt{K}=x^{1/3}$ and $h=th^2x$). The overall theory of flows in such layers (in connection with the theory of filtration) is developed recently in the works by O. V. Golubevoy and her colleagues (K. N. Bystrov, 1956-1966; Yu. A. Gladyshev, 1961, 1964; V. A. Yurisov, 1964; N. I. Gaydukov, 1966). In the more general case of the solution of the equations in question have the representations through series or the integrals, which generalize the appropriate representations analytic functions. Thus, for instance, if is written the first of equations (6.6) in the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=-\frac{\partial \ln p}{\partial x}u-\frac{\partial \ln p}{\partial y}v,$$

then these equations can be considered as system of Carleman for the generalized analytic function q(z)=u-iv (z=x+iy) (I. N. Wekua, 1959), which for a lattice obtains the representations

$$q(z)=\frac{1}{2\pi i}\oint_C q(\zeta)\,\mathrm{cth}\,\frac{\pi}{t}(\zeta-z)\,d\zeta-\frac{1}{2\pi}\iint_S(\bar{c}q+c\bar{q}-i\Omega)\,\mathrm{cth}\,\frac{\pi}{t}(\zeta-z)\,d\xi\,d\eta, \tag{6.8}$$

$$q(z)=F(z)\exp\left[-\frac{1}{2\pi}\iint_S(\bar{c}q+c\bar{q}-i\Omega)\frac{1}{q}\,\mathrm{cth}\,\frac{\pi}{t}(\zeta-z)\,d\xi\,d\eta\right]. \tag{6.9}$$

In the written formulas the integrals are taken on duct/contour C of basic airfoil/profile and on field S of determination q in the fundamental period of lattice, $c=-{}^1/_2(\partial\ln\rho/\partial x+i\partial\ln\rho/\partial y)$; F(z) - the arbitrary analytic function z.

With the aid of the representaticns indicated the methods of calculation of plane flcw (appropriate c=0) are generalized to the case of flow in the layer of the alternating/variable thickness of the incompressible fluid, and also gas (at subscnic speeds), if is utilized the method of successive approximations of ceyli - Yantsen's type. Calculations substantially become complicated due to the more complex form of basic elementary flows and need for calculating integrals according to area; therefore, the kncwr works are limited by the common/general/total discussions the application/uses of a method of special feature/peculiarities in incompressible flow (S. V. Vallander, 1958; A. M. Gokhman and E. V. N. Rao, 1965) and by solutions (vortex/eddy method) direct and reverse/inverse problems in the simplest cases h=x (I. A. Simonov, 1950, 1957), and $h=x^{-2}$ (N. G. Belekhova, 1958; K. A. Kiselev, 1958; B. S. Raukhman, 1965), and also, by the construction of elementary flows from the lattice of sources in layer $h=x^{2n}$ (Yu. A. Gladyshev, 1964) and the lattice of dipoles in layer h=exp μx (V. A. Yurisov, 1964). For the calculation of the flows of gas within limits of vane channels, are developed and are virtually applied the simpler numerical and approximation methods; of them, itself idle time it is instituted on the averaging of flow across channel (on y) and the information of task to one-dimensional (G. Yu. Stepanov, 1962; V. I. Mitrokhin, 1966).

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Mentioned "channel" methods, mention their simplicities, have still the advantage that they without essential difficulties are spread to the cases of the variable across channel values of the stagnation parameters and mixed (sub- and supersonic) flows.

As a whole solution of spatial problem as totalities of two described two-dimensional tasks at present is substantially moved gives sufficient basis/bases for reliable calculation and the designs of turbomachines. However, the complete solution of problem requires the account of all rejected members of equations, including transverse composing velocities, or, in other words, the estimation of the distortion of axisymmetric stream surfaces. This estimation can be conducted either by the method of solution of complete equations or (which more corresponds to the basic idea of the separation of spatial problem to two-dimensional ones) via the examination of the third and last/latter task of the so-called secondary flows in flow cross sections. The first theoretical explanation secondary flows as of consequence of rotation of retained vortex lines was given in 1914 to N. E. Joukowski, in the examination by it of the motion of alluviums during the rotation of river.

Secondary flows were studied, furthermore, by N. E. Joukowski and S. A. Chaplygin in flow with free vortices after screw/propeller and after finite-span wing, and also A. A. Friedman, by N. E. Kochin and I. A. Kibelem in the tasks of dynamic meteorology. Theoretical studies of secondary flows in the vane channels of turbomachines is based on the common/general/total theorems about the motion of eddy/vortices. Most essential results were obtained by V. R. Khautorn (Proc. Cambridge Phil. Soc., 1955, 51:4, 737-743). Fundamental equation for the calculation of secondary eddying along flow line in motionless channel can be written in the form

$$\rho v \frac{d}{ds}\left(\frac{\Omega_s}{\rho v}\right) = \frac{2K_s}{\rho^* v}\frac{dp^*}{dn}, \qquad (6.10)$$

where p^* and ρ^* - stagnation parameters, K_s - the geodetic curvature of flow line on the surface of the Bernoullis (p^*=const), the derivatives d/ds and d/dn are taken respectively along flow line and along the normal to Bernoulli's surface. From equation (6.10) it is evident that the development of secondary flows in nonviscous liquid is determined only by total-pressure gradient and by the curvature of flow line. In relative motion in the rotating channel to values Ω_s is added another projection on the flow line of relative eddy/vortex -2ω). Obtained values Ω_s determine the velocity field in the cross section of channel and respectively the distortion of the form of the initial axisymmetric surfaces of Bernoulli, and also average stream surfaces (and, in particular, the flow exit angles from lattices).

Further refinement of setting and solution of spatial problem occurs in the direction of the refinement of the models of flow taking into account the effects of imperfect gas, first of all to viscosity. The fact is that the theory of secondary flows in nonviscous liquid qualitatively correctly describes phenomenon; however, does not explain the emergence of total-pressure gradient in the main flow and fading secondary flows, for which it is necessary to consider viscosity effect, not small near the limiting surfaces and in regions with high total-pressure gradients. It is interesting to note that N. E. Joukowski in already mentioned work (1914) gave the theory of secondary flows in viscous fluid in thin layer, valid with an accuracy to a small second order. In 1935, P. A. Walter, it investigated in detail the developed secondary flow of viscous fluid in bent tube of round cross-section.

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Turbulent flow for a long time did not yield to theoretical studies and they were the object/subject only of experimental study. The early experimental data on secondary flows in ducts and channels in 1935 were generalized by G. N. Abramovich. The first investigations of secondary flows in turbomachines were connected with the determination of the secondary losses of kinetic energy (in comparison with ideal flow). One of the first detailed experimental

investigations of secondary cascade flows belongs to M. Ye. Deutsch, who in 1945, establish/installed the helical character of the motion of liquid near blade tips and zone of the increased losses, not depending on the length of blades. In the subsequent numerous experimental investigations was more precisely formulated three-dimensional/space flow pattern and were investigated the dependences of the coefficient of secondary losses and lattices on different geometric and hydrodynamic parameters (E. A. Gukasova, 1954, 1960, 1964; M. E. Deutsch and G. S. Samoylovich, 1959; L. G. Naumova and G. Yu. Stepanov, 1954, 1962; A. S. Ginevskiy and S. A. Dovzhik, 1959, 1961; A. V. Kolesnikov, 1959, 1963). Theoretical studies of the secondary flows of viscous fluid were developed in our country in essence in the direction of development and use of theory of three-dimensional boundary layer along the intersection of two planes (L. G. Loytsyanskiy, 1941), with external downwash (V. V. Struminskiy, 1946, 1956), on the rotating bodies (L. A. Dorfman, 1956-1960), on motionless wall with transverse pressure gradient (L. G. Naumova, 1957; V. V. Bogdanov, 1960, 1965; V. G. Pavlov, 1961; G. Yu. Stepanov, 1958, 1962). The calculations of three-dimensional boundary layer on the end-type walls of vane channels made it possible to satisfactorily explain the observing in experiments properties of secondary flows and to rate/estimate the values of secondary losses (N. M. Markov, 1958; G. Yu. Stepanov, 1958, 1962; A. V. Kolesnikov, 1964, etc.).

On the basis of given survey/coverage the development of the hydrodynamic cascade theory it is possible to break into four basic stages: (I) setting and the solution of the first problems for the lattice of plates; (II) the development of the overall cascade theory from fine/thin airfoil/profiles; (III) the complete solution of straight line and inverse problems in plane flow with the subsequent account to compressibility and the viscosity of liquid and their use in the practice of calculation and shaping of the lattices of turbomachines; (IV) inversion to the contemporary problems of the unsteady and three-dimensional/space flow around lattices.

All enumerated stages Russian and Soviet scientists played the outstanding role in the setting of tasks, their solution and application/appendices. The conducted investigations and the achieved/reached results are reflected in the vast literature, which includes special monographs, and are personified in the construction/designs of gas-turbine and rocket engines, in energy and auxiliary turbomachines.

The main trends of further development of the cascade theories it is possible to consider, in the first place, improvement and expansion of the practice of the application/use of known methods

and, in the second place, the development of the problems of unsteady and three-dimensional/space flow in turbomachines (by including the problem of aeroelasticity) taking into account the properties of real liquids. The solution of the problems indicated requires the development of the new models of flow, wide use of contemporary computer technology and new methods of experimental studies.

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